Intensity Characteristics of Ground Motion Vectors

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Abstract. The Code for Seismic Design of Buildings stipulates that the ratio of combined values should be 1:0.85:0.65 when considering three-way seismic action. However, it is sometimes found that the difference between the horizontal records and the vertical records in the characteristics of ground motion and peak vibration is very different from those of the two. With the development of the seismic theory of the engineering structure, more complex structural systems have put forward more specific requirements for ground motion input. And the simple application of the three-dimensional seismic action to the combination of the 1:0.85:0.65 pairs of earthquakes is of limited applicability. In order to unify the ratio relationship of ground motion input components in multi-dimensional ground motion, the vector signal was constructed by the components of the ground motion vector. The components of the vector signal were orthogonal to each other in a complex space. So the correlation limit of bainite's theorem and Nuttall theorem was avoided when the orthogonal vector was constructed by Hilbert. At the same time, the problem of one dimension ground motion intensity characteristics to reflect the incomplete surface of the structure response was solved. Firstly, the ratio of the horizontal component and the vertical component to the three-dimensional envelope was calculated by the model fitting, and compared with the law of the traditional multi-dimensional ground motion ratio, and the reliability and practicability of the ratio rule calculated by the new method were clearly defined. Then the time-history characteristics of the ratio were obtained by the function fitting. The corresponding model parameters were obtained by the optimization algorithm and the attenuation law was studied. It is proved that the fitting model is correct and effective in the time-history analysis of ratio.

Introduction

At present, the relationship between the amplitude ratio of the three-way action component (two levels and one vertical direction) of the most commonly used ground motion is the ratio of the three ground motion level components PGA, and the relationship is 1:0.85:0.65 [1]. This ratio is related to the relative level of the intensity level of the three horizontal component strong earthquakes to a certain extent, but it is obvious that the relative heights of the three horizontal component strong earthquakes are time-dependent [2], with the seismic resistance of the engineering structure. With the development of theory, more complex structural systems put forward more special requirements for ground motion input. The amplitude ratio of ground motion horizontal component based on 1:0.85:0.65 does not fully reflect the relative intensity of three horizontal classification strong earthquakes. Relationships are not sufficient to accurately simulate seismic effects [2-3]. In addition, the ratio of PGA is only a means to characterize the intensity ratio, and the ratio of the energy contained in the three components can also be used as a means, and the physical meaning is more clear [3].

Therefore, the paper first takes the strong earthquake record as the base [4], which is based on the ratio of PGA, the ratio of acceleration segment and the ratio of energy, and then studies the sectional statistical characteristic of the ratio of ground motion intensity [5]. Secondly, based on the results of the piecewise statistical characteristics, and based on the three-dimensional vector of the ground motion, two horizontal component strength packages are proposed. A mathematical model of the ratio of the line ratio and the ratio of the vertical component to the strength of the three-dimensional strength to reveal the variation of the relative magnitude of the horizontal
component of the ground motion with the time. Finally, the attenuation of the two horizontal component intensity envelope ratios and the mathematical model of the vertical component and the three-dimensional intensity envelope ratio are given by statistical analysis. Through the attenuation law of the strength envelope ratio model, the relationship between the ratio and the magnitude, epicentral distance and characteristic period is obtained, and the ratio of the intensity of the ground motion component is obtained. By using the ratio relationship of the strength envelopes, the intensity envelope of each component is further obtained from the three-dimensional intensity envelope model of the vector ground motion, and the three-dimensional intensity envelope of the vector ground motion and the transformation of each component intensity envelope are realized.

**Statistical Characteristics of Intensity Ratio**

**Recording Situation**

Accurate simulation of ground motion characteristics depends on a large number of real seismic records. At present, some developed countries and regions in the world have collected a number of strong earthquake records. This paper widely uses seismic records from different periods around the world, which makes the distribution of samples in time and space expand, which helps to generalize the generality. The characteristics of the law, and introduced the selected ground motion database and screening principles.

In order to ensure the accuracy of the simulation in this paper, this paper collects detailed seismic records, recorded from the Pacific Earthquake Engineering Research Center (PEER) NGA database [6]; European Strong-Motion Database (European Strong-Motion Database [7]; Japan's National Research Institute for Earth Science and Disaster Prevention (NIED) [8], New Zealand's Geonet program [9]. Most of them are derived from the NGE database of PEER.

The NGA database is currently the most representative and the most widely used database. The Pacific Earthquake Engineering Research Center and other organizations have conducted the Next Generation Attenuation to develop ground-ground attenuation relationships for the United States. To provide researchers with a database of records, NGA has collected a number of databases of different earthquakes around the world.

A three-dimensional vector ground motion and horizontal angle \( \theta(t) \) and elevation angle are \( \gamma(t) \), which are projected onto three coordinate axes of X, Y and Z to obtain three components. Conversely, the seismic record measured from an observation point usually contains three mutually orthogonal ground motions. The three ground motions can be used as orthogonal components to construct a three-dimensional vector ground motion, which is an analytical signal with clear physical benefits. The use of ground motion component orthogonality and physical meaning to construct analytical signals and analyze the description of ground motion has the significant advantage that the physical meaning is more intuitive and clear when describing certain parameters.

**PGA Ratio Statistical Analysis**

The whole peak acceleration (PGA) ratio of the vector ground motion is statistically analyzed to investigate the relationship between the amplitude characteristics of the vector ground motion. The PGA ratio of the horizontal ground component of the vector ground motion and the PGA ratio of the vertical component to the three-dimensional vector ground motion are mainly calculated. The statistical results are shown in the table 1. It can be seen that if the ratio of 1:0.85:0.65 is specified according to the specification, the ratio of the vertical component to the three-dimensional vector should be 0.44, and the statistical value is 0.42, which is slightly smaller than the normative result. The horizontal component statistic is 0.7818, which is smaller than the norm ratio.
Table 1. Ratio of PGA.

<table>
<thead>
<tr>
<th>Statistical value</th>
<th>Mean value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal component</td>
<td>0.7818</td>
<td>0.1509</td>
</tr>
<tr>
<td>Vertical and three-dimensional</td>
<td>0.4202</td>
<td>0.1798</td>
</tr>
</tbody>
</table>

**Segmentation Statistics of Acceleration Ratio**

**Horizontal Component Strength Envelope Ratio.** For the constructed analytical signal, the ratio of the two horizontal component accelerations is obtained to obtain the ratio of the intensity envelopes at each moment, and the upper envelope of the horizontal component intensity envelope ratio is obtained by modeling. Finally, through the median value of the upper envelope of the ratio, a curve with practical significance is obtained. The upper bound of the upper envelope indicates the occurrence of all the maximum ratios, the lower bound indicates the range in which the maximum ratio may occur, and the curve composed of the median indicates the most likely occurrence of the maximum ratio at each moment, representing the horizontal component strength pack. The variation of the line ratio over time (as shown in Figure 1).

![Figure 1. Horizontal component strength envelope ratio.](image-url)

According to the rising section, the platform section and the descending section of the strength envelope, the envelope of the ratio is divided into three sections correspondingly to analyze the attenuation law. The attenuation law of the mean and standard deviation of the upper envelope in the first, second and third segments with the epicenter distance \( R \) and the magnitude \( M \) is not obvious with the attenuation of the epicentral distance and magnitude. Therefore, assuming that the ratio distribution is a lognormal distribution, the 84th percentile is obtained from equations (1)-(3), and the total mean of the two statistics, the mean value and the standard deviation, are obtained, and the statistical law of the ratio of the intensity envelope line of the horizontal component is quantitatively expressed. The statistical law of the line ratio is calculated according to the three segments (as shown in Table 1). It can be seen that from the first segment to the third segment, the mean value of the envelope gradually increases, but the increase is not significant. The overall mean fluctuates around 0.9. The average value of the second segment (corresponding to the strong earthquake segment) is 0.9002. It is larger than the 0.85 ratio in the specification. The variance of the envelope from the first to the third ratio gradually decreases, and the degree of dispersion of the ratio decreases.

\[
\theta_m = \exp\left(\frac{1}{n} \sum_{i=1}^{n} \ln y_i \right) \quad (1)
\]

\[
\beta = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\ln y_i - \ln \theta_m)^2} \quad (2)
\]
\[ y_{84\text{th}} = \theta_m \cdot e^\beta \]  

Table 1. Horizontal component strength envelope ratio.

<table>
<thead>
<tr>
<th>Segmentation statistics</th>
<th>Mean value</th>
<th>Standard deviation</th>
<th>84 percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first paragraph</td>
<td>0.8948</td>
<td>0.0478</td>
<td>0.9293</td>
</tr>
<tr>
<td>The second paragraph</td>
<td>0.9002</td>
<td>0.0468</td>
<td>0.9373</td>
</tr>
<tr>
<td>The third paragraph</td>
<td>0.9138</td>
<td>0.0431</td>
<td>0.9553</td>
</tr>
</tbody>
</table>

**Vertical Component and 3D Strength Envelope Ratio.** As in the method of 1.1, the statistical acceleration ratio obtains the ratio of the vertical component to the moment of the three-dimensional vector intensity envelope, and obtains the upper envelope of the vertical component and the three-dimensional vector intensity envelope ratio by modeling, and passes the ratio. The median value of the upper envelope represents the variation of the vertical component and the three-dimensional intensity envelope ratio with time (Figure 2).

![Figure 2. Vertical component and vector intensity envelope ratio.](image)

According to the rising section, the platform section and the descending section of the strength envelope, the vertical component is divided into three sections corresponding to the upper envelope of the three-dimensional intensity envelope ratio to analyze the attenuation law. The mean and standard deviation are not obvious in the first, second and third stages with the attenuation distance R and the magnitude M. Therefore, the average of the two statistics and the 84th percentile of the mean and standard deviation are used to quantify the statistical law of the ratio of the vertical component to the three-dimensional vector intensity envelope, and the statistics are performed according to the three segments (eg: Table 2). It can be seen that from the first segment to the third segment, the mean value of the intensity envelope ratio decreases, showing a downward trend. The average value of the second segment (corresponding to the platform segment) is 0.6084, which is larger than the 0.44 value obtained according to the specification 1:0.65. From the first segment to the third segment, the ratio gradually increases, showing an upward trend, and the ratio dispersion is reduced. That is, when the earthquake starts, the vertical component first reaches the structure and the vertical component dominates. Then the horizontal component also acts on the structure. As time goes by, the vertical component decreases, and the horizontal component increases as the main component of the main ground motion.

Table 2. Vertical component and vector intensity envelope ratio.

<table>
<thead>
<tr>
<th>Segmentation statistics</th>
<th>Mean value</th>
<th>Standard deviation</th>
<th>84 percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first paragraph</td>
<td>0.8273</td>
<td>0.0848</td>
<td>0.9456</td>
</tr>
<tr>
<td>Second paragraph</td>
<td>0.6084</td>
<td>0.1399</td>
<td>0.7333</td>
</tr>
<tr>
<td>Third paragraph</td>
<td>0.5872</td>
<td>0.1411</td>
<td>0.7084</td>
</tr>
</tbody>
</table>

**Statistical Analysis of Energy-Based Intensity Ratios- Segmentation Statistics of Root Mean Square Acceleration**

Since the root mean square acceleration is positively correlated with the cumulative energy, the ratio of the root mean square acceleration of the ground motion vector can represent the intensity ratio of
the ground motion energy. According to this principle, the square and acceleration ratios of the
ground motion vectors are statistically analyzed, so that the ground level intensity ratio is
represented by the energy level and compared with the PGA ratio. In the statistical process,
statistical analysis is carried out according to the three sections of the ascending section, the
platform section and the descending section of the strength envelope.

**Horizontal Component Mean Square and Acceleration Statistics.** The horizontal component
mean square and acceleration are:

\[ \alpha_{mh} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \alpha_{hi}^2} \]  \hspace{1cm} (4)

The accelerations of the two horizontal components are calculated and compared, and the
statistical results are shown in Table 1.3. The ratio of the horizontal component in the specification
is 1:0.85, and the ratio of the horizontal component of the mean square to the acceleration method
corresponds to the second segment of the intensity envelope (the strong earthquake) is 0.8379. The
results obtained by the method of strength relation are very close. However, the energy level can
more intuitively reflect the relative high and low correlation of the ground motion component in the
intensity level of the strong earthquake section, and the physical meaning is more clearly. At the
same time, it can be seen from the statistical results that from the first segment to the third segment,
the mean value of the root mean square acceleration ratio of the two horizontal components is
increasing and the standard deviation is decreasing, and the corresponding strength envelope rise,
platform and descending segment, and ground motion horizontal component can be obtained. The
intensity slightly increases and the degree of dispersion decreases.

<table>
<thead>
<tr>
<th>Piecewise statistics</th>
<th>Mean value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first paragraph</td>
<td>0.8153</td>
<td>0.1087</td>
</tr>
<tr>
<td>Second paragraph</td>
<td>0.8379</td>
<td>0.09190</td>
</tr>
<tr>
<td>The third paragraph</td>
<td>0.8750</td>
<td>0.06856</td>
</tr>
</tbody>
</table>

**Vertical component and 3D strength envelope line mean square and acceleration statistics.**
The vertical component mean square and acceleration are:

\[ \alpha_{mv} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \alpha_{vi}^2} \]  \hspace{1cm} (5)

The three-dimensional vector mean square and acceleration are:

\[ \alpha_m = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\alpha_{h1i}^2 + \alpha_{h2i}^2 + \alpha_{vi}^2)} \]  \hspace{1cm} (6)

The vertical root mean square acceleration is compared with the three-dimensional vector root
mean square acceleration. The statistical results are shown in Table 4. When the ground motion
intensity is expressed by the PGA ratio, the vertical component ratio is 1:0.65. According to the
corresponding relationship of the three-dimensional vector ratio, the square of the ratio is obtained.
The ratio of the ground motion intensity expressed by the PGA ratio to the energy level is 1:0. 1970.
It can be seen that the ratio of the mean square of the horizontal component to the platform segment
(second segment) of the acceleration corresponding to the intensity envelope is 0.1146, which is
smaller than the ratio of the horizontal component represented by 0.1970 in the PGA ratio,
indicating that the ground motion is reflected from the energy level. The vertical component and the
three-dimensional vector intensity ratio are smaller than the PGA ratio in the intensity level of the
strong seismic segment, and the traditional PGA ratio method is more conservative. It can be seen
that, corresponding to the rising section (second section) of the seismic intensity envelope, the ratio
is significantly larger than the other section ratios and the degree of dispersion is large, which
indicate that the corresponding strength envelope is rising, and the vertical component intensity has
a greater influence on the overall strength. It is unstable and the intensity of the vertical component
tends to be stable in and after the strong earthquake section.
Table 4. Mean-to-acceleration ratio of vertical component and three-dimensional vector.

<table>
<thead>
<tr>
<th>Piecewise statistics</th>
<th>Mean value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First paragraph</td>
<td>0.6261</td>
<td>0.1900</td>
</tr>
<tr>
<td>The second paragraph</td>
<td>0.3240</td>
<td>0.1017</td>
</tr>
<tr>
<td>The third paragraph</td>
<td>0.3446</td>
<td>0.0935</td>
</tr>
</tbody>
</table>

**Ratio Model Based on Three-dimensional Vector of Ground Motion**

Through the ratio of the vertical component to the three-dimensional vector intensity envelope, the intensity envelope of the vertical component and the two-dimensional intensity envelope ratio of the two horizontal components can be obtained from the three-dimensional vector intensity envelope. Then, from the ratio relationship of the two horizontal component intensity envelopes, the intensity envelopes of the two horizontal components can be further obtained separately. Therefore, the comparison value line is fitted to obtain the parameter and time-course law of the ratio line model, which is of great significance for the transformation of the three-dimensional intensity envelope and the intensity envelope between each component.

For the horizontal component two-dimensional intensity envelope ratio, according to the trend of the trend line, the fitting function model (7) is used to fit. The fitting results are shown in Figure 3.

\[
a + (1 - a) \cdot e^{-(bx)}
\]  

For the vertical component and the three-dimensional intensity envelope ratio, according to the change trend of the ratio line, the function model (8) is used for fitting. In order to facilitate the representation and analysis of the parameters, the above function model is written as the formula (9). The fitting results are shown in Figure 2.1(b).

\[
(1 - b) + ab \cdot e^{-(cx)}
\]

\[
a + b \cdot e^{-(cx)}
\]

![Figure 3. Ratio fitting function model.](image)

**Statistical Analysis of Model Parameters**

**Horizontal Component Ratio**

For the horizontal component two-dimensional intensity envelope ratio fitting model, the regression coefficients obtained by the correlation regression model function (10) are shown in Table 5.

\[
F(x) \sim X_1 + X_2 M + X_3 R + X_4 \log T_g
\]  

(10)
Table 5. Parameters correspond to the coefficients of the regression function.

<table>
<thead>
<tr>
<th>Regression coefficient</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>-5.369×10^{-2}</td>
<td>-1.865</td>
</tr>
<tr>
<td>X2</td>
<td>5.229×10^{-4}</td>
<td>3.646×10^{-2}</td>
</tr>
<tr>
<td>X3</td>
<td>-1.79×10^{-5}</td>
<td>3.646×10^{-2}</td>
</tr>
<tr>
<td>X4</td>
<td>6.371×10^{-3}</td>
<td>1.903</td>
</tr>
</tbody>
</table>

Vertical Component to Three-dimensional Vector Ratio

For the vertical component and the three-dimensional intensity envelope ratio fitting model, the regression coefficients obtained by the correlation regression model function (11) are shown in Table 6. It can be seen from the distribution map that the parameters $a$ and $b$ are poorly correlated with the magnitude, the correlation for the epicentral distance is not obvious, and the correlation is good for the characteristic period. In addition, the coefficient $b$ has a concentrated distribution at a value of 0. In this case, the fitting function of the vertical component and the three-dimensional intensity envelope ratio is a fixed value, and the value is constant as a coefficient, which indicates that there is a vertical in the ground motion.

$$F(x) \sim X_1 + X_2M + X_3R + X_4\log T_g$$ (11)

Table 6. Parameters correspond to the coefficients of the regression function.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>-0.3817</td>
<td>-1.443</td>
<td>0.1688</td>
</tr>
<tr>
<td>X2</td>
<td>6.366×10^{-3}</td>
<td>0.1420</td>
<td>-0.1130</td>
</tr>
<tr>
<td>X3</td>
<td>-5.054×10^{-4}</td>
<td>1.0743×10^{-3}</td>
<td>-3.920×10^{-3}</td>
</tr>
<tr>
<td>X4</td>
<td>-0.2330</td>
<td>0.5985</td>
<td>-7.263×10^{-2}</td>
</tr>
</tbody>
</table>

By the regression function (11), by determining a single variable, a fitting model of the vertical component and the three-dimensional intensity envelope ratio can be obtained. When the other two quantities are fixed, one of the variables will change following the magnitude $M$, the epicentral distance $R$, and the characteristic period $T_g$ respectively (as shown in Figure 7). The relationship between coefficients and seismic conditions and the variation of coefficients between coefficients can be seen more intuitively. At the same time, the coefficient of the other two ground motion parameters can be determined, and the ratio of the vertical component to the three-dimensional intensity envelope is obtained according to the ratio fitting model.

In addition, according to the regression coefficient in Table 6, under the condition that the ground motion parameters $M$, $R$, $T_g$ are known, the correlation coefficient of the vertical component to the three-dimensional intensity envelope ratio fitting model can be obtained by the regression function (12), which is directly obtained. Ratio fit function.

For example, under the condition of magnitude $M=6$, epicentral distance=100m, and characteristic period $T_g=0.5s$. The coefficient $a=0.4993$, $b=0.4270$, $c=0.3836$, and obtain the horizontal component two-dimensional intensity envelope ratio fitting model function:

$$F(t) = 0.4993 + 0.4270 \cdot e^{-(0.3836x)}$$ (12)

This method can also be used in the horizontal component ratio in 3.1.
Conclusion

In this paper, the strength characteristics of vector ground motion and its modeling and laws are studied. The main conclusions are as follows.

1) The method of using the acceleration ratio segmentation statistics and the mean square and acceleration is proposed. The ratio of the combined value from the time to the energy level to the traditional seismic action is compared with the specification requirement of 1:0.85:0.65, which shows the acceleration. The ratio segmentation statistical method and the mean square and acceleration method are segmented and analyzed by the actual meaning of the ground motion vector, which more fully characterizes the ground motion component intensity relationship.

2) Based on the three-dimensional vector of ground motion, two mathematical models of the intensity ratio of the horizontal component and the ratio of the vertical component to the three-dimensional intensity envelope are proposed, revealing the relative variation of the intensity of the ground motion component with time. Through the attenuation law of the strength envelope ratio model, the correlation between the ratio and the magnitude, epicentral distance and characteristic period is obtained.

3) Combined with the research in this paper, the fitting function of the ratio can be directly obtained by fitting the correlation coefficient of the model to the M, the correlation between R and Tg. The proportional relationship between the horizontal component and the vertical component and the three-dimensional vector is obtained, and the relationship between the components is converted according to the three-dimensional vector ground motion relationship, and the components are obtained from the three-dimensional vector.

References


