A Robust Joint Estimation Method of Time Delay and Doppler Frequency Shift

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Abstract. In this paper, a novel concept, the Sigmoid transform based cyclic correlation, is proposed and a relative estimator named the Sigmoid transform based cyclic cross-ambiguity function is also derived to handle the joint estimation problem in the presence of impulsive noise and co-channel interference. Based on these definitions, a novel joint estimation method of the time delay and the Doppler frequency shift is proposed. Simulations have verified its superior performances over existing methods based on cyclic ambiguity function or generalized fractional lower-order cyclic cross-ambiguity function, especially under impulsive noise. Meanwhile, it is more applicable for practical applications since it is parameter-free.

Introduction

During the past decades, specialists and scholars have made huge contributions to the research on the cross ambiguity function (CAF) [1-3], especially in the presence of Gaussian noise. A linear canonical transform was introduced to ambiguity function for the time-frequency analysis of the cubic phase signal in [4]. A generalized Wigner–Ville distribution (WDL) was combined with CAF in [5]. However, these methods are derived upon the second-order statistics, which leads to their deterioration under impulsive noise environment. To better the solution, a full set of novel concepts and methods are proposed and developed after the proposal of the fractional lower order statistics by Nikias and the fractional lower order statistics based ambiguity function[6; 7](FLOSAF) is one of the methods. FLOSAF works well under both Gaussian and non-Gaussian noises, but its performances deteriorates in the presence of co-channel interference. On the other hand, although cyclic ambiguity function (CCA) proposed in [8] could handle the co-channel interference, it is still a second-order statistics based method. Meanwhile, it deteriorates under impulsive noise undoubtedly. During the search for a better method which considers both Non-Gaussian noise model and co-channel interference, the fractional lower-order cyclic cross-ambiguity function (FCCA) and the generalized fractional lower-order cyclic cross-ambiguity function algorithm (GFCCA) are presented in [9]. However, they still have their own shortcomings. For instance, these two methods are based on the fractional lower-order statistics (FLOS), which leads to their reliance on the a priori knowledge of impulsive noise during the choices of the order parameters a and b. Moreover, the performance of FLOS-based methods will degrade under highly impulsive noise. Therefore, a void still remains in the search for a novel method with better robustness in the presence of both non-Gaussian noise and co-channel interference.

In this paper, a novel concept termed the Sigmoid transform based cyclic correlation (SigmoidCC) is proposed and a relative estimator named Sigmoid transform based cyclic cross-ambiguity function (SigmoidCCA) is presented to fulfill the needs mentioned above. Simulations have verified its
superiority over existing methods under both non-Gaussian noise and co-channel interference. In this paper, the non-Gaussian noise is modeled by alpha-stable distribution[10].

**Background**

**Signal Model**

Suppose two received signals \( x(t) \) and \( y(t) \) as

\[
x(t) = s(t) + w_1(t) + s_i(t)
\]

\[
y(t) = s(t - D) e^{-j2\pi ft} + w_2(t) + s_i(t)
\]

where \( s(t) \) denotes the signal of interest and it is a cyclostationary signal. \( D \) denotes the time delay between the receivers. \( f_d \) denotes the Doppler frequency shift caused by the relative moving between the receiver and the object. \( s_i(t) \) denotes the co-channel interference modulated with the same carrier frequency with \( s(t) \). \( w_1(t) \) and \( w_2(t) \) denote additive noises which obey alpha-stable distribution[11]. As a direct generalization of the Gaussian distribution, the alpha-stable distribution is completely determined by four parameters: the characteristic exponent \( \alpha \), the symmetric parameter \( \beta \), the location parameter \( \mu \) and the dispersion parameter \( \gamma \).

**SigmoidCCA**

Sigmoid transform is a commonly used nonlinear transform[12-14]. Its definition is shown in Eq.(3).

\[
\text{Sigmoid}\left[ x(t) \right] = \frac{2}{1 + \exp\left[ -x(t) \right]} - 1
\]

For complex \( Z(t) = X(t) + jY(t) \),

\[
\text{Sigmoid}\left[ Z(t) \right] = \frac{2}{1 + \exp\left( -X(t) \right)\left( \cos(-Y(t)) + jsin(-Y(t)) \right)} - 1
\]

A novel cyclic correlation \( R_{\text{Sigmoid}}^x(\tau) \) referred to as Sigmoid transform based cyclic correlation (SigmoidCC) is defined in Eq.(5),

\[
R_{\text{Sigmoid}}^x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \text{Sigmoid}\left[ x(t + \tau/2) \right] \text{Sigmoid}^* \left[ x(t - \tau/2) \right] e^{-j2\pi \tau} dt
\]

For a finite duration signal, \( R_{\text{Sigmoid}}^x(\tau) \) could be estimated by Eq.(6).

\[
\hat{R}_{\text{Sigmoid}}^x(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} \text{Sigmoid}\left[ x(t + \tau/2) \right] \text{Sigmoid}^* \left[ x(t - \tau/2) \right] e^{-j2\pi \tau} dt
\]

Sigmoid transform based cyclic cross-correlation \( R_{\text{Sigmoid}}^{xy}(\tau) \) is given in Eq.(7).

\[
R_{\text{Sigmoid}}^{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \text{Sigmoid}\left[ x(t + \tau/2) \right] \text{Sigmoid}^* \left[ y(t - \tau/2) \right] e^{-j2\pi \tau} dt
\]

For finite duration signals, \( R_{\text{Sigmoid}}^{xy}(\tau) \) could be estimated by Eq.(8).
A novel cyclic ambiguity function $C_{yx,Sigmoid}^e(u,f)$, named Sigmoid transform based cyclic cross-ambiguity function (SigmoidCCA), is shown in Eq.(9).

$$
C_{yx,Sigmoid}^e(u,f) = \int R_{x,Sigmoid}^e(\tau + u)\left[R_{x,Sigmoid}^e(\tau)\right]^* e^{-j2\pi f \tau} d\tau
$$

And the joint estimation method based on SigmoidCCA is shown in Eq.(10)

$$
\left(\hat{D}, \hat{f}_d\right) = \arg\max \left|C_{yx,Sigmoid}^e(u,f)\right|
$$

**Simulation**

Simulation conditions: The signal of interest is set as a BPSK signal with the carrier frequency $f_c = 0.5 f_s$ and the baud rate $a_{dt} = 0.2 f_s$, where $f_s$ is the sampling frequency. Time delay is set as 16 sample intervals and the Doppler frequency shift (DFS) is set as $0.3125 f_s$. The co-channel interference is another BPSK signal with the same carrier frequency but a different baud rate $a_{dt} = 0.6 f_s$. The signal to interference ratio (SIO) is 0dB. The additional noises are $\alpha S\alpha$ noises with $\mu = 0$. The generalized signal-to-noise ratio (GSNR) is employed to measure the noise intensity, and it is defined in Eq.(11)

$$
\text{GSNR} = 10 \log_{10}(\sigma_s^2/\gamma_w) \text{ (dB)}
$$

where $\sigma_s^2$ is the signal variance and $\gamma$ is the dispersion parameter of $\alpha S\alpha$ noise. The estimation accuracy $P_a$ is defined in Eq.(12), where $\hat{V}$ is the estimation value and $V$ is the true one.

$$
P_a = \left(1 - \frac{\left|\hat{V} - V\right|}{V}\right) \times 100\%
$$

In order to verify the superior performances of SigmoidCCA over existing methods such as CCA, FCCA and GFCCA, simulation experiments are carried out as follows. And the order parameters for FCCA and GFCCA are $a = b = 0.1$. The results in Simulations 2 and 3 are based on 500 Monte-Carlo simulations.

Simulation 1: CCA, FCCA, GFCCA and SigmoidCCA for a single estimation

Simulation 1 shows the estimation results of CCA, FCCA, GFCCA and SigmoidCCA for a single trial of data under both $\alpha S\alpha$ noise and co-channel interference. In Figure 1, the red solid line denotes the true value of time delay or DFS.
Figure 1. The estimation results of CCA, FCCA, GFCCA and SigmoidCCAunder co-channel interference (SIO = 0dB) and impulsive noise (GSNR = 0dB, $\alpha = 1$).

In Figure 1, CCA fails when impulsive noise is presented. GFCCA also fails under intensive impulsive noise as $\alpha = 1$. On contrast, SigmoidCCA has clear peaks. Meanwhile, cyclostationary theory is utilized in all the methods, which keeps 3D plots from wrong peaks caused by co-channel interference and verifies their robustness for co-channel interference.

Simulation 2: estimation accuracy versus GSNR

From Figure 2, when GSNR is higher than 10dB, the four methods all work well under weak impulsive noise. CCA deteriorates quickly when GSNR decreases. SigmoidCCA outperforms other methods when GSNR is extremely low.

Simulation 3: estimation accuracy versus characteristic exponent $\alpha$ (GSNR = 0dB)

Simulation 3 is the comparison among four methods when GSNR = 0dB and the results are shown in Figure 3. Similar to Simulation 2, all these methods perform well if noise obeys Gaussian distribution when $\alpha = 2$. On the other hand, when the noise becomes more impulsive, for example when $\alpha < 1$, the proposed SigmoidCCA has the best estimation accuracy.
Conclusion

In this paper, a novel concept named Sigmoid transform based cyclic correlation (SigmoidCC) is first proposed. Then a robust estimator termed Sigmoid transform based cyclic cross-ambiguity function (SigmoidCCA) is derived upon SigmoidCC to handle the time delay and Doppler frequency shift problem in the presence of impulsive noise and co-channel interference. Theoretical analysis and simulation results show that SigmoidCCA has better estimation precision than the others, such as CCA, FCCA and GFCCA methods, even when GSNR is extremely low or when the impulsiveness is intensive.

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References


