Equilibrium Customer Strategies in Markovian Queues with Working Vacations and N-policy

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Abstract. In this paper, we consider the equilibrium of the customers in an M/M/1 queueing model with working vacations and N-policy. This paper assumed the server rate turn to a low rate when the number of customers in the system is below N and a work vacation begin. The server service rate switch to high rate only if the number of customers in the queue is more than N when a vacation ends. With considering waiting cost and reward, the equilibrium strategies of a customer are derived under the conditions that the system is fully observable and fully unobservable. The effect of parameters on the equilibrium threshold are presented by numerical examples.

Introduction

Recently, the economic analysis of queueing systems has been studied extensively. For the works studying on customer's equilibrium balking strategies in classical vacation queueing models, initially, Burnetas and Economou [1] presented several Markovian queues with setup times, and they analyzed the customer's balking strategies under the four precision levels of system information. Economou and Kanta [2] studied the equilibrium balking strategies in the observable queue with breakdowns and repairs. Li, Wang and Zhang [3] dealt with equilibrium threshold strategies in Markovian queues with partial breakdowns for fully observable case and fully unobservable case, respectively.

Different from the classical vacation policies discussed in the literature above, working vacation is one kind of policy under which the server can provide the service at a lower speed during the vacation period rather than stopping completely, and was introduced by Servi and Finn [4]. They first studied the M/M/1 queueing system with working vacations, and obtained the transform formulae for the distribution of the number of customers in the system and sojourn time in steady state. Wu and Takagi extended Servi and Finn's [5] M/M/1/WV model to an M/G/1/WV model, where the service times during service period, the service times during working vacation as well as vacation times are generally distributed. Baba [6] developed a GI/M/1 queue with multiple working vacations. Li, Tian and Ma [7] analyzed the GI/M/1 queue with working vacations and vacation interruption. Using the matrix analysis method, the main steady-state performance measures such as mean queue length and waiting time are obtained. Li and Cheng [8] considered two M/M/1 queues with working vacations and two policy: m-policy and (n, N)-policy, respectively. Recently, Zhang et al. [9] studied the equilibrium balking strategies in Markovian queues with working vacations under different information levels. Sun and Li [10] analyzed the socially optimal balking strategies in queues with multiple working vacations. Sun and Li [10] presented the customers' equilibrium balking behavior in some single-sever Markovian queues with two-stage working vacations.

In the classical queue with vacation policies, there is a vacation policy named N-policy, which the customers can't make their decisions. Guo and Hassin [11] elaborately studied fully observable and unobservable queues with homogeneous and heterogeneous customers under N-policy, respectively. They assumed that the server begin a busy period as soon as there are N customers waiting in the queue during the vacation period. As a complementary work, Guo and Li [12] analyzed the same issue with [11] in some partially observable queues. Differ from the N-policy discussed by Guo et al. [11, 12], Sun and Li et al. [13] assumed that a new busy period begins only if the server finds at least
N customers waiting in the queue after completing a vacation. Dimitrakopoulos and Burnetas [14] discussed the customer equilibrium strategies in an M/M/1 queue under dynamic service control. The service rate switches between a low and a high value depending on system congestion. That is, the service rate is kept at a low value when the number of customers in the system is at or below a threshold level T, and turns to a high value when the system congestion is above T.

The present paper aims to study the equilibrium of the customers in an M/M/1 queueing model with working vocations and N-policy. This paper assumed the server rate turn to a low rant when the number of customers in the system is below N and a work vacation begin. During the work vacation, the server serves the customers at a low rate. Differ from Dimitrakopoulos and Burnetas [14], the service rate is kept at a low level during work vacations, whatever the number of customers in the system is below or above T. The server service rate switch to high rate only if the number of customers in the queue is more than N When a vacation ends. In this work we will carry out the equilibrium analysis of the system under the conditions that the system is fully observable and fully unobservable.

The paper organizes as follows. In the next section, some basic assumption and the reward-cost structure are described. In Section 3 and Section 4, we analyze the equilibrium strategies for fully observable queue and fully unobservable queue, respectively. For each type queue, we derive the customer's equilibrium balking strategies and equilibrium social welfare. We give some numerical results. The conclusion and some suggestions for future research are given in Section 5.

Description of the Model

In this paper, we consider an M/M/1 queue system with working vocations and N-policy. Assume that customer's potential arrival rate is λ and the server's service rate in regular busy period is μ₁. At the instant of a completion, the server begins a vacation of random length at the instant when the queue length is below N and vacation duration V follows an exponential distribution with parameter θ. During a vacation, the original customers or arriving customers in a vacation period can be served at a lower rate μ₀. When a vacation ends, if the number of customers in the queue is less than N, another vacation is taken; Otherwise, the server service rate from μ₀ to μ₁. So we denominate this type of queues as M/M/1/WV queues with N-threshold policy.

We represent the state at time t by the pair (N(t), I(t)), where N(t) and I(t) denote the number of customers and the state of the server (0: on working vacation; 1: regular busy period) respectively. It is clear that the process \{(N(t), I(t)), t ≥ 0\} is a two-dimensional continuous time Markov chain.

We assume that the customers are allowed to decide whether to join or balk upon their arrival based on the information they have. After service, every customer receives a reward of R units. This may reflect his satisfaction or the added value of being served. Moreover, there exists a waiting cost C units per time when the customers remain in the system including the time of waiting in the queue and being served. We adopt a linear cost function, and his/her expected net benefit after service, denoted by U, is U=R-CE(W), where E(W) represents his/her mean sojourn time. Obviously, U=0 if he balks. We assume the service discipline is first in first out, and jockeying and reneging are not allowed.

Observable Queues with N Threshold

In this section, we consider the customer’s equilibrium balking threshold at state i, which can be denoted \( n_e(i) \), and pure threshold strategy is expressed as \( (n_e(0), n_e(1)) \).

We consider the customers’ equilibrium balking behavior in the observable queues, i.e., they can observe both the state of the server I(t) and the number of the present customers N(t) at arrival at time t. We have the following result.
**Theorem 1** In the observable M/M/1/WV with N-threshold there exist thresholds \((n_e(0), n_e(1))\) which are given by

\begin{equation}
    n_e(1) = \left[ \frac{R}{C} - \frac{N-1}{\mu_0} \right] \mu_1 + N - 2,
\end{equation}

\begin{equation}
    n_e(0) = [x_e],
\end{equation}

where \(x_e\) is the unique root of the equation

\begin{equation}
    \frac{x-x_0-2}{\mu_1} + \frac{N-1}{\mu_0} + \frac{1}{\theta} \left( 1 - \frac{\mu_0}{\mu_1} \right) \left( 1 - \left( \frac{\mu_0}{\mu_0+\theta} \right)^x \right) = \frac{R}{C}.
\end{equation}

**Proof.** Let \(W(n, i)\) be the sojourn time of the marked customer in case he/she encounters the system state \((n, i)\). Then, we have the system

\begin{equation}
    E[W(n, 1)] = \frac{n-N}{\mu_1} + E[W(N, 1)], \quad n \geq N + 1,
\end{equation}

\begin{equation}
    E[W(N, 1)] = \frac{1}{\mu_1} + E(W_N),
\end{equation}

where \(W_N\) denotes the sojourn time of the marked customer when he/she encounters the system state \((N - 1, 1)\), and \(E(W_N) = \frac{1}{\mu_1} + E[W(N - 2, 0)]\).

\begin{equation}
    E[W(n, 0)] = \frac{n-N-2}{\mu_1}, \quad 0 \leq n \leq N - 2,
\end{equation}

\begin{equation}
    E[W(N - 1, 0)] = \frac{1}{\theta - \mu_0} + \frac{\mu_0}{\theta - \mu_0} E[W(N - 2, 0)] + \frac{\theta}{\theta + \mu_0} E(W_N),
\end{equation}

\begin{equation}
    E[W(N, 0)] = \frac{1}{\theta + \mu_0} + \frac{\mu_0}{\theta + \mu_0} E[W(n - 1, 0)] + \frac{\theta}{\theta + \mu_0} E[W(n, 1)], \quad n \geq N.
\end{equation}

Taking (4) and (5) into (3), we obtain

\begin{equation}
    E[W(n, 1)] = \frac{n-N-2}{\mu_1} + \frac{N-1}{\mu_0}.
\end{equation}

By iterating (7) and taking into account (6) and (8), we have

\begin{equation}
    E[W(n, 0)] = \frac{n-N-2}{\mu_1} + \frac{N-1}{\mu_0} + \frac{1}{\theta} \left( 1 - \frac{\mu_0}{\mu_1} \right) \left( 1 - \left( \frac{\mu_0}{\mu_0+\theta} \right)^x \right).
\end{equation}

Solving \(U_e(n, 1) = 0\), we can get

\begin{equation}
    n_e(1) = \left[ \frac{R}{C} - \frac{N-1}{\mu_0} \right] \mu_1 + N - 2.
\end{equation}

To ensure the server can always be reactivated, we assume throughout the paper that \(R > C\left( \frac{N}{\theta + \mu_0} + \frac{\theta}{\mu_1} \left[ \frac{1}{\mu_1} + \frac{N-1}{\mu_0} \right] \right)\).

If \(n \leq N\), we can easily check that \(E(W(n, 0))\) is strictly increasing for \(n\). Solving \(U_e(n, 0) = 0\) we can get \(n_e(0)\).
Fig.1-Fig.3 shows that the relation $n_e(1) \geq n_e(0)$ always maintains when $\mu_0 < \mu_1$, which is obviously. Under varying, $n_e(i)(i = 0, 1)$ all decrease with $N$. $n_e(1)$ is fixed and $n_e(0)$ is increase with $\theta$, respectively. Fig.2-Fig.3 depict the equilibrium thresholds versus service rate $\mu_1$ and $\mu_0$.

Next we will derive the stationary queue length distribution in observable queues. In view of the relations of $n_e(0) \leq n_e(1)$, the state space of the Markovian process $\{N(t), I(t)\}$ is

$$\Omega_{ob1} = \{(n,0): 0 \leq n \leq n_e(0) + 1\} \cup \{(n,1): 0 \leq n \leq n_e(1) + 1\}.$$

Denote the stationary queue length distribution as

$$\pi_{n,j} = P\{N = n, I = j\} = \lim_{t \to \infty} P\{N(t) = n, I(t) = j\}, \ (n,j) \in \Omega_{ob1}.$$

Figure 3. $n_e$ vs $\mu_1$ in an observable queue.

Then the stationary transition probability equations can be written as

$$\lambda \pi_{0,0} = \mu_0 \pi_{1,0},$$

$$(\lambda + \mu_0) \pi_{n,0} = \mu_0 \pi_{n+1,0} + \lambda \pi_{n-1,0}, \ 1 \leq n \leq N - 2,$$

$$(\lambda + \mu_0) \pi_{N-1,0} = \mu_0 \pi_{N,0} + \lambda \pi_{N-2,0} + \mu_1 \pi_{N,1},$$

$$(\lambda + \mu_0 + \theta) \pi_{n,0} = \mu_0 \pi_{n+1,0} + \lambda \pi_{n-1,0}, \ N \leq n \leq n_e(0),$$

$$(\mu_0 + \theta) \pi_{n_e(0)+1,0} = \lambda \pi_{n_e(0)+1,0},$$

$$(\lambda + \mu_1) \pi_{N,1} = \mu_1 \pi_{N+1,1} + \theta \pi_{N,0},$$

$$(\lambda + \mu_1) \pi_{n,1} = \mu_1 \pi_{n+1,1} + \lambda \pi_{n-1,1} + \theta \pi_{n,0}, \ N + 1 \leq n \leq n_e(0) + 1,$$

$$(\lambda + \mu_1) \pi_{n,1} = \mu_1 \pi_{n+1,1} + \lambda \pi_{n-1,1}, \ n_e(0) + 2 \leq n \leq n_e(1),$$

$$\mu_1 \pi_{n_e(1)+1,1} = \lambda \pi_{n_e(1)+1,1}.$$ (18)

For the working vacation state, we first consider the probabilities $\{\pi_{n,0}\}$ for $n \leq N - 1\}$. From (11), we notice that they are the solutions of the following homogeneous linear difference equations with constant coefficients

$$\mu_0 x_{n+1} - (\lambda + \mu_0) x_n + \lambda x_{n-1} = 0, 1 \leq n \leq N - 2.$$ (19)

So its corresponding characteristic equations is

$$\mu_0 x^2 - (\lambda + \mu_0)x + \lambda = 0,$$

which has two roots: 1 and $\rho_0 = \lambda/\mu_0$. So the general solution of (19), denoted by $x_{n}^{hom}$, is

$$x_{n}^{hom} = A_1 + B_1 \rho_0^n,$$

where $A_1, B_1$ are the coefficients to be determined. From (10), we have
which yields

\[
\begin{align*}
A_1 + B_1 &= \pi_{0,0}, \\
\lambda(A_1 + B_1) &= \mu_0(A_1 + B_1\rho_0).
\end{align*}
\]

(20)

Therefore,

\[
\pi_{n,0} = \rho_0^n \pi_{0,0}, \quad 0 \leq n \leq N - 1,
\]

(21)

where \(\pi_{0,0}\) will be given in the following paragraph.

Then we consider the probabilities \(\{\pi_{n,0}|N \leq n \leq n_\rho(0) + 1\}\). From (13), they are the solution of the following homogeneous linear difference equation with constant coefficients

\[
\mu_0 x_{n+1} - (\lambda + \theta + \mu_0) x_n + \lambda x_{n-1} = 0, N \leq n \leq n_\rho(0).
\]

(22)

Then general solution of (22) is \(x_n^{\text{hom}} = A_2 x_1^{*n} + B_2 x_2^{*n}\), where

\[
x_{1,2}^{*} = \frac{\lambda + \mu_0 + \theta + \sqrt{(\lambda + \mu_0 + \theta)^2 - 4\lambda\mu_0}}{2\mu_0}
\]

and \(A_2, B_2\) are to be determined. Taking into account (12), (14) and (21), then obtain

\[
\begin{align*}
A_2 &= \frac{-\mu_0(\lambda + \mu_0)x_1^{*} - \lambda}{x_2^{*N-1}[(\theta + \mu_0)x_1^{*} - \lambda][\lambda + \mu_0 - \lambda_0 - \mu_0 x_2^{*}] - x_1^{*N-1}[(\theta + \mu_0)x_2^{*} - \lambda][\lambda + \mu_0 - \lambda_0 - \mu_0 x_1^{*}]}, \\
B_2 &= \frac{-\mu_0(\lambda + \mu_0)x_2^{*} - \lambda}{x_2^{*N-1}[(\theta + \mu_0)x_1^{*} - \lambda][\lambda + \mu_0 - \lambda_0 - \mu_0 x_2^{*}] - x_1^{*N-1}[(\theta + \mu_0)x_2^{*} - \lambda][\lambda + \mu_0 - \lambda_0 - \mu_0 x_1^{*}]}, \\
\pi_{0,0} &= \frac{A_2 x_1^{*N-1} + B_2 x_2^{*N-1}}{\rho_0^{N-1}}.
\end{align*}
\]

(23)

So we get

\[
\pi_{n,0} = \pi_{0,0}^n \rho_0^n, 0 \leq n \leq N - 1,
\]

(24)

\[
\pi_{n,0} = A_2 x_1^{*n} + B_2 x_2^{*n}, N \leq n \leq n_\rho(0) + 1,
\]

(25)

where \(\pi_{0,0}, A_2, B_2\) are given by (23).

Sequentially analyzing the server’s busy state, we first derive the probabilities \(\{\pi_{n,1}|N \leq n \leq n_\rho(0) + 1\}\). From (16), they are the solutions of the following nonhomogeneous linear difference equation

\[
\mu_1 x_{n+1} - (\lambda + \mu_1) x_n + \lambda x_{n-1} = -\theta(A_2 x_1^{*n} + B_2 x_2^{*n}), N + 1 \leq n \leq n_\rho(0) + 1,
\]

(26)

The general solution of the homogeneous version of (26) is \(x_n^{\text{hom}} = A_3 + B_3 \rho^n\). Hence the general solution of (26), denoted by \(x_n^{\text{gen}}\), is given as \(x_n^{\text{gen}} = x_n^{\text{hom}} + x_n^{\text{spec}}\), where \(x_n^{\text{spec}}\) is a specific solution of (26). We consider a specific solution of the form \(x_n^{\text{spec}} = C_1 x_1^{*n} + D_1 x_2^{*n}\). Substituting it into (26), we get

\[
\begin{align*}
C_1 &= \frac{-\theta A_2 x_1^{*}}{\mu_1 x_1^{*2} - (\lambda + \mu_1)x_1^{*} + \lambda}, \\
D_1 &= \frac{-\theta B_2 x_2^{*}}{\mu_1 x_1^{*2} - (\lambda + \mu_1)x_1^{*} + \lambda},
\end{align*}
\]

(27)

Thus,

\[
x_n^{\text{gen}} = A_3 + B_3 \rho^n + C_1 x_1^{*n} + D_1 x_2^{*n}, N \leq n \leq n_\rho(0) + 1
\]

(28)

where \(A_3, B_3\) is to be determined. Taking into account (15), then obtain

\[
\begin{align*}
A_3 &= \frac{\mu_1 \pi_{N,1} - \theta A_2 + \mu_1 x_1^{*} - \lambda]}{\mu_1 (1 - \rho)^N} \left[ x_1^{*N} + \theta B_2 + D_1 (x_2^{*} - 1) \right] x_2^{*N}, \\
B_3 &= \frac{\lambda \pi_{N,1} - \theta A_2 + C_1 \mu_1 x_1^{*} - \lambda]}{\mu_1 (1 - \rho)^N} \left[ x_1^{*N} + \theta B_2 + D_1 (x_2^{*} - 1) \right] x_2^{*N},
\end{align*}
\]

(29)

thus, from (28),
\[ \pi_{n,1} = A_3 + B_3 \rho^n + C_1 x_1^n + D_1 x_2^n, N \leq n \leq n_e(0) + 1. \]  
(30)
can be obtained, where \( C_1, D_1, A_3, B_3 \) are given by (27) and (29).

Finally, we consider the probabilities \( \{ \pi_{n,1} \mid n_e(0) + 2 \leq n \leq n_e(1) + 1 \} \). From (17), we get \( x_{n}^{\text{hom}} = A_4 + B_4 \rho^n \), where \( A_4, B_4 \) are to be determined. Taking into account (18) and (28), we obtain \( A_4 = 0 \) and

\[ B_4 = \frac{A_3 + B_3 \rho^{n_e(0) + 1} + C_1 x_1^{n_e(0) + 1} + D_1 x_2^{n_e(0) + 1}}{\rho^{n_e(0) + 1}}. \]  
(31)
Therefore,

\[ \pi_{n,1} = B_4 \rho^n, n_e(0) + 2 \leq n \leq n_e(1) + 1. \]  
(32)
where \( B_4 \) is given by (31).

In summary, we find that the obtained stationary state probabilities \( \{ \pi_{n,j} \mid (n, j) \in \Omega_{\text{obs}} \} \) are all related to \( \pi_{N,1} \). Using the normalization condition, we get the results in the following theorem.

\textbf{Theorem 2} For the observable Markovian queue with working vocations and N-policy and state space \( \Omega_{\text{obs}} = \{(n, 0) : 0 \leq n \leq n_e(0) + 1\} \cup \{(n, 1) : N \leq n \leq n_e(1) + 1\} \), if \( N \leq n_e(0) \leq n_e(1) \), then the stationary queue length distribution \( \{ \pi_{n,j} \mid (n, j) \in \Omega_{\text{obs}} \} \) is:

\[ \begin{cases} 
\pi_{n,0} = \pi_{0,0} \rho^n, & 0 \leq n \leq N - 1, \\
\pi_{n,0} = A_2 x_1^n + B_2 x_2^n, & N \leq n \leq n_e(0) + 1,
\end{cases} \]
and
\[ \begin{cases} 
\pi_{n,1} = A_3 + B_3 \rho^n + C_1 x_1^n + D_1 x_2^n, & N \leq n \leq n_e(0) + 1, \\
\pi_{n,1} = B_4 \rho^n, n_e(0) + 2 \leq n \leq n_e(1) + 1.
\end{cases} \]

\textbf{The Unobservable Queues}

In the unobservable queues, the customers’ decision problem is to select a joining probability \( q(0 \leq q \leq 1) \), and the effective arrival rate is \( \lambda_1 = \lambda q \). So their equilibrium mixed strategy is denoted by equilibrium joining probability \( q^* \), or by equilibrium arrival rate \( \lambda_{1,e} = \lambda q^* \). On the other hand, their socially optimal mixed strategy is denoted by optimal joining probability \( q^a \) or by optimal arrival rate \( \lambda^1 = \lambda q^a \).

We first discuss the customers’ equilibrium balking behavior in the unobservable queues, i.e., arriving customers can observe neither the server’s current state \( I(t) \) nor the system occupancy \( L(t) \) at time \( t \). In order to derive \( \lambda_{1,e} \), we first try to get the stationary system queue length distribution. We can obviously observe that \( (I(t), L(t)) \) is a quasi-birth-and-death (QBD) process with the state space

\[ \Omega = \{(k, 0) : 0 \leq k \leq N - 1\} \cup \{(k, j) : k \geq N, j = 0, 1\}. \]
If \( \rho = \frac{\lambda}{\mu_1} < 1 \), let \( (I, J) \) be the stationary limit of the QBD process \( \{I(t), J(t)\} \).

\[ \pi_{k,j} = P(I = k, J = j) = \lim_{t \to \infty} P\{I(t) = k, J(t) = j\}, (k, j) \in \Omega. \]
and the infinitesimal generator of the process as \( Q_1 \). Using the lexicographical sequence for the states, the infinitesimal generator can be written as

\[ Q_1 = \begin{pmatrix}
-\lambda & C_0 & 0 & 0 & \cdots & 0 \\
B_1 & A_1 & C_1 & 0 & \cdots & 0 \\
& B_{N-1} & A_{N-1} & C_{N-1} & \cdots & 0 \\
& & B_N & A_{N-1} & C_N & \cdots \\
& & & B & A & C \\
& & & & \cdots & \cdots & \cdots
\end{pmatrix}, \]
where

\[ B_k = \mu_0, \quad 1 \leq k \leq N - 1, \quad B_N = (\mu_0, \mu_1)^T; \]
Using the same procedure as in Li and Cheng [11], we give the following result.

**Theorem 3** If \( \rho < 1 \), the stationary probability distribution of \((Q_1, J_1)\) is

\[
\begin{aligned}
\pi_{n,0} &= K \left( \frac{\lambda_1}{\mu_0} \right)^n, 0 \leq n \leq N - 1, \\
\pi_{n,0} &= K \left( \frac{\lambda_1}{\mu_0} \right)^{N-1} \rho^{n-N+1}, N \leq n,
\end{aligned}
\]

where
\[
K = \left[ \sum_{n=0}^{N-1} \left( \frac{\lambda_1}{\mu_0} \right)^n + \left( \frac{\lambda_1}{\mu_0} \right)^{N-1} \frac{1}{1-r} + \frac{\lambda_1}{\mu_0} \right]^{-1}
\]
and
\[
r = \frac{1}{2\rho_0} \left( \lambda_1 + \theta + \mu_0 - \sqrt{\lambda_1 + \theta + \mu_0^2} - 4\lambda_1\mu_0 \right).
\]

Further, we can obtain the unconditional mean queue length, denoted by \( E(L) \), is

\[
E(L) = K \left( \frac{\rho_0}{1-\rho_0} \right) \left[ \frac{\rho_0}{1-\rho_0} - N \right] + K \rho_0 \left[ \frac{\rho_0}{1-\rho_0} - \frac{\rho_0}{1-\rho_0} \right] + K \rho_0 \left[ \frac{\rho_0}{1-\rho_0} - \frac{\rho_0}{1-\rho_0} \right]
\]
and then the expected sojourn time of a joining customer, denoted by \( E(W) \), is
\[
E(W) = \frac{E(L)}{\lambda q}.
\]

It is nearly impossible to take theoretical analysis because of complexity of the expressions. Therefore, we make some numerical experiments. Fig.4. Shows that the expected sojourn time of a customer who decides to join, is strictly increasing for \( q \in [0, 1] \). Hence, the customer's expected net benefit, denoted by \( U(q) = R - C E(W) \). We numerically observe that that \( U(q) = 0 \) has a unique unique positive root \( q^* \), and the customers' equilibrium mixed strategy is \( q_e = \min \{ q^*, 1 \} \).

In Fig.5, we observe that with the increase of threshold \( N \), the Equilibrium entrance probability \( q_e \) decrease, but the decreasing trend becomes not evident when \( N \) increases to one certain value. This can be explained in practice. The larger the threshold, the larger the number of customer in the system, so that an arriving customer has less inclined to join the system. But when the threshold achieves on certain value, the customers face more congested system and much longer waiting delay, this will cause the customer's joining trending toward stabilization.
Fig. 6-8 show the sensitivity of the equilibrium entrance probability with respect to $\theta$, $\lambda$, $\mu_0$, respectively. Evidently, along with the increase of the $\mu_0$ and $\theta$, $q_e$ increase, respectively. And, we also find that the threshold $N$ has small effect on the $q_e$, if $\mu_0$, $\theta$ are too larger. Meanwhile, with the increase of $\lambda$, $q_e$ decrease. Such change trends are consistent with the practical situations which can be simulated by the model we consider.

**Conclusion**

In this paper, we consider the equilibrium behavior, in the fully observable and fully unobservable M/M/1 queueing model with working vocations and N-policy. The effect of parameters on the equilibrium threshold are presented by numerical examples. Carried out an analysis M/M/1-G queue with working breakdowns and impatient customers. We have obtained the queue length distribution. Various performance measures such as the probability of server state, the average queue length, the sojourn time in the queue are also carried out.

For future, we can extend this paper to complex models such as queues with batch arrivals or M/G/1 case with generally distributed impatience times.

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**Reference**


