**Singular Kinetics Analysis of Cartesian Serial-Parallel Manipulator**

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**Abstract.** New type of universal Cartesian serial-parallel manipulator in singular configuration, kinetics equations and Euler’s equations constructed, complete analysis of each jointed points of the moving platform carried out, inverse kinetics solution can be obtained with the given parameters, which include the position and orientation of the manipulator, applied force on the output shaft, parameters of motion of output shaft and connecting bar and sliding block, computer programming adopted to get the valid data and its figure, thus found the perfect show of redundant driving structure on this special mechanism.

**Introduction**

Based on the traditional parallel machine tools, which constructed by Stewart platform mechanism, their working space is very narrow relative to the size of the mechanism, the mechanism are less flexible, and they have the singular configurations, the complexity of the kinematic positive solution, between the controlling of the position and the orientation have the coupling phenomenon, which results in the increase of the control difficulty\([1]\).These problems have undoubtedly affected the extension of their application to a certain extent. Therefore, based on the research and analysis of traditional series-parallel manipulators, a new type of universal Cartesian serial-parallel manipulator is proposed\(\text{as shown in Figure 1}\).It inherits the advantages of the series and the parallel mechanism\([2]\),it not only has a good control performance, but also reduces the manufacturing cost and the processing difficulty\([3]\).

![A diagram of universal Cartesian serial-parallel manipulator.](image)

The mechanism has six spatial degrees of freedom, translation along the spatial \(x\), \(y\), \(z\) axes, and the rotation of the spatial \(x\), \(y\), \(z\) axes, it controls all translational and rotational motion of the mechanism through eight linear actuated sliders, the position control and orientation control of the mechanism output platform are decoupled, and the difficulty of control and compensation motion is
simplified, the design of a new type of plane spherical hinge makes the traditional spherical hinge limited by the pendulum angle has improved, and the symmetrical mechanism makes the whole production, assembly and replaceability of the mechanism superior to the traditional Stewart platform mechanism. When the mechanism is in a singular configuration, the stress state at the hinge points on the moving platform is worth exploring. Through using multi-rigid-body theory\cite{4} of the spiral to given the singularity orientation in any given position, force and torque of the output terminal is known to reverse solving the stress of the hinge points on the moving platform, the kinetics inverse solution of the mechanism in given space position and related parameters is analyzed by establishing kinetics equations and Euler's equations\cite{5}, and an intuitive analysis and calculation result is given to the object in the form of examples.

**Singular Force Analysis of Mechanism**

When the mechanism is singular, its spatial force needs to be based on the singularity of the mechanism. Because there are two moving platforms in the upper and lower layers of the mechanism, and each layer can be regarded as a plane quadrilateral mechanism, therefore, the relevant knowledge of mechanism shows that the working space of the output spindle has changed greatly if the structure form of the mechanism meets the special form of parallelogram, as shown in Fig.2, mechanism of each layer of the parallelogram mechanism can be equivalent to a single rod mechanism with fixed hinge point and fixed length around the horizontal guide rail, and its reachable workspace is the whole area enclosed by the limit position of the slider, as shown in Figure 2, the shaded areas is enclosed by the boundaries $X_{\text{max}}$, $X_{\text{min}}$, $\text{Lim1}$, $\text{Lim2}$, but the boundary lines of the reachable workspace in the vertical direction of the mechanism are the same as those without singularities. At this time, the position of the mechanism output spindle has numerous kinds of conditions, so this is the singular configuration of the mechanism. At this point, if the mechanism is subject to the $F_m$, $M_m$, shown in Figure 3, the mechanism is not working, and the forces acting on each hinge point of the moving platform and the force acting at each input hinge point are analyzed as follows.

![Figure 2. Mechanism singular configuration and workspace sectional.](image-url)
First of all, at the hinge points $b_1$, $b_2$, $b_3$, $b_4$ of the moving platform, the revolving motion of which the connecting rods $B_1b_1$, $B_2b_2$, $B_3b_3$, $B_4b_4$ with a fixed length $l$ are taken as radius and the input hinge point $B_1$, $B_2$, $B_3$, $B_4$ on the horizontal guide rail is the center of the circle, so that we can be obtained:

$$F_{b_1}^Z = F_{b_2}^Z = F_{b_3}^Z = F_{b_4}^Z = \frac{F_{mZ}}{4}. \quad (1)$$

Thus we can know that only eight force components remain in the four hinge points $b_1$, $b_2$, $b_3$, $b_4$, and the eight force components in the direction of $x$, $y$ axes along the space coordinate coordinate system need to be solved. Suppose the distance between the upper and lower moving platform centers is $|O_1O_2| = h$, and the distance from the lower platform to the tool nose point $P_d$ is $|O_2P_d| = d_f$, the cosine angles between the spindle in the space and the directions of the $x$, $y$, $z$ axes correspond to $\alpha_1$, $\alpha_2$, $\alpha_3$, and the angle between the connecting rod and the positive direction of the horizontal guide rail is $\beta_{b_i}(i=1,2,3,4)$. First of all, according to the special form and structure of the mechanism, it can be seen that the Euler’s equation of the lower moving platform $b_3b_4$ can takes $O_2$ as a fulcrum (as shown in Figure 4) and parallels to the horizontal plane $xoy$, the equation as follows:

$$
\begin{align*}
F_{b_3}^x & | \vec{p}_b \cdot O_2 | \cos \alpha'_{b_3} + F_{b_4}^x | \vec{p}_b \cdot O_2 | \sin \alpha'_{b_4} + \\
F_{b_3}^y & | \vec{p}_b \cdot O_2 | \cos \alpha'_{b_3} + F_{b_4}^y | \vec{p}_b \cdot O_2 | \sin \alpha'_{b_4} + \\
& = J_{b_3b_4} \omega_{b_3b_4}. \quad (2)
\end{align*}
$$

Among them, $F_{b_i}^x, F_{b_i}^y, F_{b_i}^z (i=1,2,3,4)$ are the forces acting at the moving platform hinge points, which are the three force components along the $x$, $y$, $z$ directions respectively (the same below), $J_{b_3b_4}$ is a moment of inertia that the lower platform $b_3b_4$ is wound around the center point $O_2$ of the lower platform and the platform is also parallel to the OZ axis. $\omega_{b_3b_4}$ is a component of angular acceleration that the lower platform $b_3b_4$ is wound around the center point $O_2$ of the lower platform and the platform is also parallel to the OZ axis. $\alpha'_{b_3b_4}$ is the angle between the space vector $b_3b_4$ and the Y axis of the fixed coordinate system.
Similarly, taking the O\textsubscript{2} point as the fulcrum (as shown in Figure 5), the output spindle O\textsubscript{1}O\textsubscript{2} is listed around the X, Y, and Z axes respectively, and their Euler’s equations are as follows:

\[
\begin{align*}
F_x \cdot \left| P_{o1} \right| \cdot \cos \alpha_x + F_y \cdot \left| P_{o1} \right| \cdot \cos \alpha_y + M_x + \\
F_y \cdot \left| P_{o1} \right| \cdot \cos \alpha_y + F_z \cdot \left| P_{o1} \right| \cdot \cos \alpha_z + M_y + \\
F_z \cdot \left| P_{o1} \right| \cdot \cos \alpha_z + F_x \cdot \left| P_{o1} \right| \cdot \cos \alpha_x + M_z + \\
J_{o2} \cdot \dot{\omega}_{o2} = J_x \cdot \dot{\omega}_{o2} \\
J_y \cdot \dot{\omega}_{o2} = J_y \cdot \dot{\omega}_{o2} \\
J_z \cdot \dot{\omega}_{o2} = J_z \cdot \dot{\omega}_{o2}
\end{align*}
\]  
(3)

\[
\begin{align*}
F_x \cdot \left| P_{o2} \right| \cdot \cos \alpha_x + F_y \cdot \left| P_{o2} \right| \cdot \cos \alpha_y + M_x + \\
F_y \cdot \left| P_{o2} \right| \cdot \cos \alpha_y + F_z \cdot \left| P_{o2} \right| \cdot \cos \alpha_z + M_y + \\
F_z \cdot \left| P_{o2} \right| \cdot \cos \alpha_z + F_x \cdot \left| P_{o2} \right| \cdot \cos \alpha_x + M_z + \\
J_{o2} \cdot \dot{\omega}_{o2} = J_x \cdot \dot{\omega}_{o2} \\
J_y \cdot \dot{\omega}_{o2} = J_y \cdot \dot{\omega}_{o2} \\
J_z \cdot \dot{\omega}_{o2} = J_z \cdot \dot{\omega}_{o2}
\end{align*}
\]  
(4)

\[
\begin{align*}
F_x \cdot \left| P_{o2} \right| \cdot \cos \alpha_x + F_y \cdot \left| P_{o2} \right| \cdot \cos \alpha_y + M_x + \\
F_y \cdot \left| P_{o2} \right| \cdot \cos \alpha_y + F_z \cdot \left| P_{o2} \right| \cdot \cos \alpha_z + M_y + \\
F_z \cdot \left| P_{o2} \right| \cdot \cos \alpha_z + F_x \cdot \left| P_{o2} \right| \cdot \cos \alpha_x + M_z + \\
J_{o2} \cdot \dot{\omega}_{o2} = J_x \cdot \dot{\omega}_{o2} \\
J_y \cdot \dot{\omega}_{o2} = J_y \cdot \dot{\omega}_{o2} \\
J_z \cdot \dot{\omega}_{o2} = J_z \cdot \dot{\omega}_{o2}
\end{align*}
\]  
(5)

Among them, P\textsubscript{o1} is the tool nose point at the end of the output spindle, F\textsubscript{m} are the external forces at the tool nose point of front-end of spindles, and the F\textsubscript{x}, F\textsubscript{y}, F\textsubscript{z} are the three components of F\textsubscript{m} along the x, y, z directions respectively. M\textsubscript{m} are the external torque at the tool nose point of front-end of spindles, and the M\textsubscript{x}, M\textsubscript{y}, M\textsubscript{z} are the three components of M\textsubscript{m} along the x, y, z directions respectively. J\textsubscript{o1}, J\textsubscript{o2}, J\textsubscript{o3} are the moments of inertia that the spindle O\textsubscript{1}O\textsubscript{2} is wound around the point O\textsubscript{2} and the spindle is also parallel to the x, y, z axes. \(\omega_{o1}, \omega_{o2}, \omega_{o3}\) are the components of angular acceleration that the spindle O\textsubscript{1}O\textsubscript{2} is wound around the point O\textsubscript{2} and the spindle is also parallel to the x, y, z axes. \(\alpha_{o1}, \alpha_{o2}, \alpha_{o3}\) and \(\alpha_{o1}', \alpha_{o2}', \alpha_{o3}'\) are the angles between the space vectors \(\overrightarrow{b_{o1}}, \overrightarrow{b_{o2}}\) and the x, y, z axes of the fixed coordinate system.

![Figure 5. Torque equilibrium analysis of the spindle with O2 as a fulcrum.](image)

At the same time, each fixed length connecting rod takes each hinge point B\textsubscript{1}, B\textsubscript{2}, B\textsubscript{3}, B\textsubscript{4} as the fulcrum in the horizontal plane, and obtains four Euler’s equations as follows:

\[
\begin{align*}
F_{b1} \cdot l \cdot \sin \alpha_{b1} + F_{b1} \cdot l \cdot \cos \alpha_{b1} = J_{b1} \cdot \dot{\omega}_{b1} \\
F_{b2} \cdot l \cdot \sin \alpha_{b2} + F_{b2} \cdot l \cdot \cos \alpha_{b2} = J_{b2} \cdot \dot{\omega}_{b2} \\
F_{b3} \cdot l \cdot \sin \alpha_{b3} + F_{b3} \cdot l \cdot \cos \alpha_{b3} = J_{b3} \cdot \dot{\omega}_{b3} \\
F_{b4} \cdot l \cdot \sin \alpha_{b4} + F_{b4} \cdot l \cdot \cos \alpha_{b4} = J_{b4} \cdot \dot{\omega}_{b4}
\end{align*}
\]  
(6)

(7)

(8)
$$F_b^Y \cdot l \cdot \sin \alpha_b^{Y} + F_b^X \cdot l \cdot \cos \alpha_b^{Y} = J_{b_i b_i} \cdot \dot{\omega}_{b_i}.$$  \tag{9}

Among them, $\alpha_b^{b_i}$, $\alpha_b^{b_i}$, $\alpha_b^{b_i}$, $\alpha_b^{b_i}$ are the angles between each fixed length connecting rod and the Y axis of the fixed coordinate system. $J_{b_i b_i}, J_{b_i b_i}, J_{b_i b_i}, J_{b_i b_i}$ are the rotational inertia of each fixed length connecting rod $b_i b_i, b_i b_i, b_i b_i, b_i b_i$ around the input hinges point $b_i, b_i, b_i, b_i$ in the horizontal plane. $\dot{\omega}_{b_i}, \dot{\omega}_{b_i}, \dot{\omega}_{b_i}, \dot{\omega}_{b_i}$ are the components of angular acceleration of each fixed length connecting rod $b_i b_i, b_i b_i, b_i b_i, b_i b_i$ around the input hinges point $b_i, b_i, b_i, b_i$ in the horizontal plane.

From the formula (2) to (9), eight equations can be obtained, and we can obtain eight unknown components $F_b^X, F_b^X, F_b^X, F_b^X, F_b^Y, F_b^Y, F_b^Y, F_b^Y$ except $F_b^Z, F_b^Z, F_b^Z, F_b^Z$, thus, the spatial forces acting at each hinge point of the mechanism on the moving platform in a singular configuration can be obtained. Then the matrix expression form can be obtained:

$$J \cdot X = b.$$ \tag{10}

The force Jacobi matrix $J$ in equation (10) is the matrix that consists of the coefficients of eight unknown variables in formula (2) to (9), and $X$ is a column vector consisting of variables $[F_b^X, F_b^X, F_b^X, F_b^X, F_b^Y, F_b^Y, F_b^Y, F_b^Y, F_b^Z, F_b^Z, F_b^Z, F_b^Z], b$ is a column vector consisting of a known item that does not contain eight unknown variables in the formula (2) to (9). When the mechanism is given a spatial position and orientation, the external force $F_\omega$ of the output spindle and the external torque of a couple $M_\omega$ are all known, and the Jacobi matrix $J$ is nonsingular matrix, we can obtain a uniquely definite solution.

**Case Calculation**

The value of the fixed parameter in the known mechanism is assumed to be $I = 0.5m$, $|b_1 O_1| = |b_2 O_2| = 0.2m$, $|b_1 O_1| = |b_2 O_1| = 0.24m$, $|b_1 O_1| = 0.3m$, $|b_2 O_2| = 0.15m$, the positive angle between the connecting rods $b_1 b_1, b_2 b_2, b_3 b_3, b_4 b_4$ and the Y axis of the fixed coordinate system (that is the horizontal guide rail) is $60^\circ$. It is assumed that the mechanism is in parallelogram shape, and the coordinates of point $B_1$ is $(0,0,2000,0.6000)$, point $B_2$ is $(0.6000,0.4000)$, point $B_3$ is $(0.1600,0.1500)$, point $B_4$ is $(0.6400,0.1500)$. And the mass of the connecting rod is $m_{b_i} = 1.0kg$ ( $i = 1, 2, 3, 4$ ), the angular acceleration of the connecting rod $b_i b_i, b_i b_i, b_i b_i$, $b_i b_i$ in the horizontal plane around the input hinge points $b_i, b_i, b_i, b_i$ are $0.5rad/s^2$ , the angular acceleration of the lower moving platform around the point $O_1$ and parallel to the Z axis is $0.2rad/s^2$, the mass of the output spindle is $m_{b_i} = 1.5kg$, the angular acceleration of the output spindle around the axis of output spindle perpendicular to horizontal plane is $0.2rad/s^2$, the acting force $F_m = 100-200N$ (the step size is 1), $M_m = 100Nm$, and the direction of action is $[1 \ 1 \ 1]$. The force at each hinge point can be obtained by programming and drawing, and as shown in Figure VI. (Note: When the value of each line vector is the same as that of the corresponding coordinate positive axis, it is the positive value, otherwise it is the negative value.)

It can be seen from the Fig. 6 that the force at the two hinge points $b_i, b_2$ on the upper moving platform is consistent. The variation of the force component in the three directions at the two hinge points $b_i, b_2$ on the upper moving platform varies along the spatial fixed slope line with the change of the external force and moment, as shown in Figure 6 (a) and (b). And the force at the two hinge points $b_i, b_2$ on the upper moving platform is also consistent, the forces acting on the X and Y axes at the two hinge points on the lower moving platform keep constant with the change of the external force and moment, and the variation of the force component in the Z axis varies along the fixed proportional relationship with the change of the external force and torque, as shown in Figure 6 (c) and (d). Then it can be seen from the above that the forces at the four hinge points are not completely consistent, but there is a certain regularity.
Conclusion

Based on the advantages and disadvantages of traditional serial and parallel machine tools, a Cartesian serial-parallel manipulator is constructed, it has a good comprehensive performance, but because of the special structure of the mechanism, it has a singular configuration. In order to fully understand the forces acting on each hinge point of the moving platform in the singular state of the mechanism, in this paper, by using the Newton-Euler’s method of kinetics inverse solution analysis, and the results of analysis and deduction are computed by computer programming, it gives the force of each hinge point of the input under the given condition, so through the further analysis, we can see as follows: the magnitude and change regulation of the upper and lower moving platform are different when the output spindle is subjected to the external force and the external torque of a couple, and the force regularity at the hinge point on the same platform is the same. The hinge point of the lower moving platform has the sensitivity of three directions of X, Y and Z axes to the change of the force, while the hinge point of the upper platform has only one direction sensitivity to the Z axis.

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Reference


