Research on a Rumor Transmission Model with Refutation Mechanism

Ming-juan SUN

Yanan University, School of Mathematics and Computer Science, Yan’an 716000, China

Keywords: Propagation of rumor, Refutation mechanism, Stability, Bendixson-Dulac criteria.

Abstract. With the refutation mechanism supplemented to the SIR rumor spreading model, a modified rumor transmission model is established, which includes the process of spreader and ignorant converting to stifler when they contact. A detailed qualitative analysis about existence of its solutions and the local asymptotic stability of its equilibria are carried out. Using the theory of limit systems and the Bendixson-Dulac criteria, we obtained the sufficient conditions for the global asymptotic stability of the equilibria. A numerical simulation is carried out to illustrate the feasibility of our main results. The results show that besides of controlling the propagation threshold of rumors, controlling refutation rate and termination rate is also an effective means to restrain the rumor propagation.

Introduction

Rumors spread through social networks but that may be based on inaccurate descriptions of problems or events\cite{1}. Rumors often arise and spread with emergencies, and it is easy to cause a negative impact on public safety, therefore, it is of great theoretical value and practical significance to study the mechanism and prevention strategies of rumor propagation, and it is an important issue in the public safety crisis management. The study of the spreading of rumor has attracted the attention of scholars in sociology, communication, political science, psychology, management science and mathematics. The spreading of rumor is in many ways similar to the spreading of epidemic infection, and a classic rumor model is the DK model proposed by Daley and Kendal in 1965\cite{2}. In recent years, various factors are considered in the modelling of rumors, and various models have been established and analyzed to illustrate the rumor propagation rules under the influence of different factors, for example, network structure\cite{3}, forgetting and memory mechanisms\cite{4}, suspicion mechanism\cite{5}, counterattack mechanism\cite{6} and refutation mechanism\cite{7}, and so on.

Rumor refutation is the process of clarifying the facts such that rumors will be eliminated by the end of the process. When rumors appeared, government agencies announced the truth to the public through government radio, official Weibo and WeChat. The rumor propagation process is affected by the rumor refutation, and it also presents different evolution laws. Therefore, the rumor spreading model with refutation mechanism can be used to describe the spread of rumors more accurately. Huo and Huang\cite{8} proposed a dynamic model for unconfirmed information spreading to depict the impact of media coverage and science education on the transmission dynamics of information. Through the analysis of the impact of the government’s refutation strategies on the process of rumor spreading, Wang et al.\cite{9} established a rumor spreading model considering refutation mechanism and obtained the critical thresholds. Song and Chen\cite{10} built a rumor propagation model and mean-field equations after considering rumor refutation of public and feedback mechanism. The above literature effectively depicts the mechanism of rumor propagation, and proves that rumor spreading model is an effective tool to reveal the law of rumor propagation in different ways and provides theoretical support for the prevention and control of rumors. However, the subjective initiative of people and the impact of the rumor refutation are not fully considered. For example, ignorant and spreader will become stifler after receiving information of rumor refutation. In addition, since knowing the truth of the rumor, the stifler will become the ‘terminator’ of rumor in the process of rumor spreading, that is, the ignorant have an opportunity to contact with the stifler and know the rumor truth, so as to have a chance to directly change into the stifler. In this paper, considering the influence of rumor refutation and the propagation
rules that the ignorant turns into the stifler when the stifler clarify the truth to the ignorant, a rumor transmission model with refutation mechanism based on SIR epidemic models is proposed and the dynamic analysis is given to illustrate the rumor spreading mechanism.

This paper is organized as follows. In section 2, a modified model for rumor spreading with the refutation mechanism is proposed. In section 3, some preliminary results are given. In section 4, we obtain sufficient conditions for the global asymptotical stability of the system. In section 5, a numerical simulation is carried out to illustrate the feasibility of our main results. Finally, a brief conclusion is given in the last section.

**A Dynamical Model for Rumor Spreading**

In this section, we propose a modified model for rumor spreading with the refutation mechanism. As the schematic model flowchart is depicted in Figure 1, the transmission model is given by the following deterministic system of nonlinear ordinary differential equations:

\[
\begin{align*}
    \frac{dS(t)}{dt} &= \Lambda - \alpha \langle k \rangle S(t)I(t) - \beta \langle k \rangle S(t)R(t) - \gamma_1 S(t) - \mu S(t), \\
    \frac{dI(t)}{dt} &= \alpha \langle k \rangle S(t)I(t) - \lambda \langle k \rangle I(t)(I(t) + R(t)) - \gamma_1 I(t) - \mu I(t), \\
    \frac{dR(t)}{dt} &= \beta \langle k \rangle S(t)R(t) + \lambda \langle k \rangle I(t)(I(t) + R(t)) + \gamma_1 S(t) + \gamma_2 I(t) - \mu R(t),
\end{align*}
\]

where the ignorant, spreader, and stifler are denoted by \( S(t), I(t) \) and \( R(t) \), respectively. \( \Lambda \) is the rate at which individuals are recruited into the population, \( \mu \) is the rate at which people leave the population, \( \alpha \) is the transmission rate, \( \lambda \) is the immune rate, \( \beta \) is the termination rate, \( \gamma_1 \) and \( \gamma_2 \) are the rumor rejection rate, \( <k> \) is the average degree of network parameters.

By practical meaning, the initial conditions of (1) are given as

\[
S(0) \geq 0, \quad I(0) \geq 0, \quad R(0) \geq 0.
\]

**Preliminary Analysis**

In this section, the basic results on the boundedness of positive solutions and the existence of equilibria are presented.

**Theorem 3.1** The solution \((S(t), I(t), R(t))\) of system (1) with initial condition (2) is existent and non-negative on \([0, +\infty)\), and the set \( \Omega \) is positively invariant with respect to (1) and attracts any solution of (1), where

\[
\Omega = \left\{(S, I, R) \mid S \geq 0, I \geq 0, R \geq 0, S(t) + I(t) + R(t) \leq \frac{\Lambda}{\mu} \right\}
\]

**Proof.** Using the same method as [11], the results are obtained easily.

For the existence of the equilibria, it has the following result.
Theorem 3.2 (i) System (1) always has a boundary equilibrium $E_0 \left( S^0, 0, \frac{\Delta}{\mu} - S^0 \right)$, where
\[
S^0 = \frac{\beta \langle k \rangle \Lambda}{\mu} + \gamma_1 + \mu - \frac{\left( \frac{\beta \langle k \rangle \Lambda}{\mu} + \gamma_1 + \mu \right)^2 - 4 \beta \langle k \rangle \Lambda}{2 \beta \langle k \rangle} ;
\]
(ii) If $R_0 > 1$ and $\alpha > \beta$, there exists a single positive equilibrium, denoted by $E^+ \left( S^*, I^*, R^* \right)$, where
\[
R_0 = \frac{(\alpha + \lambda) \langle k \rangle S^0 \lambda + (\gamma_2 + \mu) \mu}{\frac{\beta \langle k \rangle \Lambda}{\mu} + \gamma_2 + \mu} , \quad S^* = \frac{\lambda \langle k \rangle \Lambda + (\gamma_2 + \mu) \mu}{(\lambda + \alpha) \langle k \rangle \mu} , \quad I^* = \frac{\Lambda - \beta \langle k \rangle \left( \frac{\Delta}{\mu} - S^* \right) S^* - (\gamma_1 + \mu) S^*}{(\alpha - \beta) \langle k \rangle S^*} ,
\]
\[
R^* = \frac{\lambda \langle k \rangle \left( \frac{\Delta}{\mu} - S^* \right)^2 + \gamma_2 S^* + \gamma_3 \left( \frac{\Delta}{\mu} - S^* \right)}{\frac{\beta \langle k \rangle \Lambda}{\mu} + \gamma_2 + \mu - (\lambda + \beta) \langle k \rangle S^+} .
\]

Proof. Letting the right side of (1) equal 0, we can easily obtain the equilibria.

Stability Analysis

Now, we are in a position to perform the stability analysis of the system. From Theorem 3.1, it is enough to consider (1) on $\Omega$, and we have the following theorems.

Theorem 4.1 (i) When $R_0 < 1$, $E_0$ is locally asymptotically stable; (ii) When $R_0 > 1$, $\alpha > \beta$ and $\beta \langle k \rangle \left( S^* \right)^2 < \Lambda$, $E^+$ is locally asymptotically stable.

Proof. The associated characteristic equation of (1) at $E_0$ is given by
\[
(x + \mu) (x - \beta \langle k \rangle S^0 + \mu + \beta \langle k \rangle R^0 + \gamma_1) (x - \alpha \langle k \rangle S^0 + \lambda \langle k \rangle R^0 + \gamma_2 + \mu) = 0 ,
\]
and the characteristic roots are
\[
x_1 = -\mu < 0 , \quad x_2 = \beta \langle k \rangle S^0 - \mu - \beta \langle k \rangle R^0 - \gamma_1 = -\frac{\left( \frac{\beta \langle k \rangle \Lambda}{\mu} + \gamma_1 + \mu \right)^2 - 4 \beta \langle k \rangle \Lambda}{2 \beta \langle k \rangle} < 0 ,
\]
\[
x_3 = \alpha \langle k \rangle S^0 - \lambda \langle k \rangle R^0 - \gamma_2 - \mu = (\alpha + \lambda) \langle k \rangle S^0 - \frac{\beta \langle k \rangle \Lambda}{\mu} - \gamma_2 - \mu ,
\]
therefore, the stability is determined by $x_1$, (i) is obtained.

The associated characteristic equation of (1) at $E^+$ is given by
\[
(x + \mu) \left( x + \alpha \langle k \rangle S^0 - \mu + \beta \langle k \rangle R^0 + \gamma_1 + (\lambda + \alpha) \langle k \rangle \left( I^* + R^0 \right) + \mu + \gamma_1 \right) = 0 ,
\]
and the characteristic roots are
\[
x_1 = -\mu < 0 , \quad x_2 \text{ and } x_3 , \quad \text{where } x_2 \text{ and } x_3 \text{ satisfy the following equation}
\]
\[
x^2 + (\alpha \langle k \rangle I^* - \beta \langle k \rangle S^0 + \mu + \beta \langle k \rangle R^0 + \gamma_1) x + (\alpha - \beta) \langle k \rangle S^* (\lambda + \alpha) \langle k \rangle I^* = 0 ,
\]
when $R_0 > 1$, $\alpha > \beta$ and $\beta \langle k \rangle \left( S^* \right)^2 < \Lambda$, the roots of (5) are negative, therefore (ii) is true.

Theorem 4.2 (i) When $R_0 < 1$, $E_0$ is globally asymptotically stable; (ii) When $R_0 > 1$, $\alpha > \beta$ and $\beta \langle k \rangle \left( S^* \right)^2 < \Lambda$, $E^+$ is globally asymptotically stable.

Proof. Let $z(t) = \frac{\Lambda}{\mu} - S(t) - I(t) - R(t)$, system (1) is equivalent to
\[\begin{aligned}
\frac{dz}{dt} &= -z, \\
\frac{dl}{dt} &= \alpha \langle k \rangle \left( \frac{\Lambda}{\mu} - z - I - R \right) I - \lambda \langle k \rangle I (I + R) - \gamma_2 I - \mu I, \\
\frac{dR}{dt} &= \beta \langle k \rangle \left( \frac{\Lambda}{\mu} - z - I - R \right) R + \lambda \langle k \rangle I (I + R) + \gamma_1 \left( \frac{\Lambda}{\mu} - z - I - R \right) + \gamma_2 I - \mu R,
\end{aligned}\]
\[(6)\]
the solutions of (6) are all in the limit set \( z = 0 \), i.e., \( \lim_{t \to \infty} (S(t) + I(t) + R(t)) = \frac{\Delta}{\mu} \). On the plane \( S + I + R = \frac{\Delta}{\mu} \), the solutions of (6) satisfy
\[\begin{aligned}
\frac{dl}{dt} &= \alpha \langle k \rangle \left( \frac{\Lambda}{\mu} - I - R \right) I - \lambda \langle k \rangle I (I + R) - \gamma_2 I - \mu I = f, \\
\frac{dR}{dt} &= \beta \langle k \rangle \left( \frac{\Lambda}{\mu} - I - R \right) R + \lambda \langle k \rangle I (I + R) + \gamma_1 \left( \frac{\Lambda}{\mu} - I - R \right) + \gamma_2 I - \mu R = g.
\end{aligned}\]
\[(7)\]
From Theorem 3.1, \( \Omega' \) is positively invariant with respect to (7), where
\[\Omega = \left\{ (I, R) \mid I \geq 0, R \geq 0, I(t) + R(t) \leq \frac{\Lambda}{\mu} \right\} .\]
Let us choose \( D(I, R) = 1/(IR) \) as a Dulac function for (7). Hence, it has that
\[\begin{aligned}
\frac{\partial(Df)}{\partial I} + \frac{\partial(Dg)}{\partial R} &= -\alpha \langle k \rangle \frac{\lambda \langle k \rangle}{R} - \beta \langle k \rangle \frac{\lambda \langle k \rangle}{R} - \frac{\lambda \langle k \rangle}{I} - \frac{\gamma_2}{\mu} - \frac{1}{\mu R} \left( \frac{\gamma_2 I}{\mu} - \gamma_1 \right) \\
&\leq -\alpha \langle k \rangle \frac{\lambda \langle k \rangle}{R} - \beta \langle k \rangle \frac{\lambda \langle k \rangle}{R} - \frac{\lambda \langle k \rangle}{I} - \frac{\gamma_2}{\mu} - \frac{1}{\mu R} \left( \frac{\gamma_2 I}{\mu} - \gamma_1 \right) < 0.
\end{aligned}\]
\[(8)\]
It follows from Dulac criterion that there cannot exist any closed trajectories in \( \Omega' \). Furthermore, from Theorem 3.1, when \( R_0 < 1 \), \( E_0 \) is locally asymptotically stable, thus, \( E_0 \) is globally asymptotically stable. Similarly, we can get the conclusion that \( E^{+} \) is globally asymptotically stable.

**Numerical Simulations**

In this section, numerical experiments with the effects of parameter changes in the proposed model are given. The results are created using MATLAB and reported as follows.

Figure 2 has been obtained for the following parameter values: \( \Lambda = 0.4 \), \( \alpha = 0.3 \), \( \beta = 0.2 \), \( \lambda = 0.2 \), \( \gamma_1 = 0.001 \), \( \gamma_2 = 0.0005 \) and \( \mu = 0.4 \), and it demonstrates the global asymptotical stability of system (1) about \( E_0 \). The global asymptotical stability of system (1) about \( E^{+} \) with the following parameters values is shown in Figure 3: \( \Lambda = 0.4 \), \( \alpha = 1 \), \( \beta = 0.1 \), \( \lambda = 0.1 \), \( \gamma_1 = 0.05 \), \( \gamma_2 = 0.01 \) and \( \mu = 0.4 \).
To explore the effects of $\gamma_1$ and $\gamma_2$ on the rumor transmission, we choose $\Lambda = 0.4$, $\alpha = 0.3$, $\beta = 0.2$, $\lambda = 0.2$, $\mu = 0.4$ and different values of $\gamma_1$ and $\gamma_2$. Figure 4 shows that the maximum density of spreaders increases with decreasing rumor rejection rates. Since the maximum spreader density equals the highest density of people who are spreading the rumor, it can be used to measure the maximum rumor influence, we can easily see from Figure 4 that the higher the rumor rejection rate, the smaller the maximum influence of a rumor.

To explore the effects of $\beta$, the parameters values are chosen as $\Lambda = 0.4$, $\alpha = 1$, $\gamma_1 = 0.3$, $\gamma_2 = 0.15$, $\lambda = 0.1$ and $\mu = 0.4$. Moreover, different values of $\beta$ are used. Figure 5 displays that the higher the termination rate, the smaller the maximum influence of a rumor.

Conclusions

Considering the influence of rumor refutation and the propagation rules that the ignorant turn into the stifler when the stifler clarify the truth to the ignorant, a rumor transmission model with refutation mechanism is proposed. Detailed analysis on global asymptotic stability of the equilibria is given. The results show that the rumor disappears when certain condition ($R_0 < 1$) is met. However, when $R_0 > 1$, $\alpha > \beta$ and $\beta < k > (S^*)^< < \Lambda$, rumor spreads. $R_0$ is defined as the threshold of rumor transmission, so controlling the size of $R_0$ is an important technical index to prevent and control rumors. Besides of controlling the propagation threshold of rumors, controlling refutation rate and termination rate is also an effective means to restrain the rumor. Therefore, to make full use of the media such as government radio, official micro-blog, and WeChat is an effective way to curb rumor spreading. Strengthening citizens' quality education and enhancing the sense of responsibility of the whole nation to resist rumors are important means to suppress rumor spreading.
Acknowledgement
This work was financially supported by the Founds of YAU (YD2015-10).

References