Spare Parts Configuration of Phased Array Radar Based on Cannibalization

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Abstract. Aiming at the characteristic of batch replacement and batch sending to repair for the large phased array radar’s spare parts, a configuration model of LRU and SRU based on cannibalization is established. Firstly, this paper selected three kinds of LRU and the SRU which is subordinate to the LRU as the research object, broke the traditional constraint of unlimited maintenance capacity at the depot-level, then established a spares of LRU and SRU configuration model that minimizes the acquisition cost of system under the restriction of the expected fill rate of LRU and SRU, lastly marginal efficiency analysis algorithm was applied to achieve the specific parameters. The living examples show that the model can optimize the parameters including number of spare parts, feasibility of cannibalization and replacement rate of SRU, which will improve the supporting effectiveness of large phased array radar.

Introduction

Spare parts play an important role in the equipment support, so how to optimize the spare parts configuration and improve the availability of equipment has become a hot issue for many scholars. For some equipment that performs specific combat mission, due to the limited maintenance capability and stock of spare parts, and there is a certain distance between combat mission location and supply location, cannibalization strategy is widely adopted and used in the equipment maintenance support, its characteristic is that for the failure line replaced unit(LRU), when the stocks of shop replaced unit(SRU) are low, concentrate the failure SRU on the fewer LRU, which will reduce the maintenance latency time caused by the shortage of SRU. Such as large phased array radar, warships equipment, and aircrafts that perform tasks.

The study of spare parts configuration based on cannibalization at present is mainly focused on the following two aspects [1-5]: (1) the spare parts configuration based on cannibalization with single failure component being sent to repair; (2) the spare parts configuration based on incomplete cannibalization. But literature above are all under the precondition that failure component is sent to repair singly without considering the way of batch failure modules being sent to repair, in order to solve the problems, this paper puts spare parts configuration of phased array radar as an example, and establishes a spare parts configuration model under the two-level maintenance with the queue theory that minimizes the acquisition cost of system under the restriction of the expected fill rate of LRU and SRU.

Problem Description and Critical Assumption

Problem Description

The typical phased array radar’s structure is shown in Figure 1. It is assumed that a phased array radar is made up of three kinds of LRU (T/R modules, power units, composition units), LRU is composed of various SRU, and the equipment failure is caused by the failure of SRU. Each LRU of the phased array radar is independent, and only when the number of functional units reaches at least \( k_i \), it can complete the specified detection tasks.
The paper adopts \( (m_i, N_i) \) maintenance strategy when at modeling, namely, when the failure number of LRU reaches \( m_i \), spare parts of LRU will be transported to the use spot from the grass-root level warehouse, if the grass-root level warehouse have spare parts of LRU, part of the failure LRU will be replaced, the rest of failure LRU is divided into two parts, one part for cannibalization, another part for replacement of failure SRU on the LRU, phased array radar equipment begins to work properly after maintain, failure SRU of replacement is sent to depot-level repair workshop, at the same time repaired SRU will be added to the grass-root level warehouse, the detailed process is shown in Figure 2.

**Critical Assumptions**

According to the two-level maintenance of phased array radar, the following critical assumptions are needed:

1. Each LRU and SRU in the phased array radar equipment are independent, and both LRU and SRU are series structure;
2. Ignoring the detection and locating time of the failure SRU on the LRU, each failure LRU is caused by only one failure SRU;
3. The life of LRU and SRU are exponentially distributed;
4. Lateral transshipment between spare parts is not considered;
5. The spare parts shall be sent in batches, and the depot-level repair workshop shall only be responsible for the maintenance of SRU.

**Expected FILL Rate Model**

**Variable description**

\( R(t) \): reliability of the system composed by LRU; \( r(t) \): reliability of single LRU; \( \lambda_i \): failure rate of the LRU; \( \lambda_y \): failure rate of SRU; \( \alpha \): feasibility of cannibalization; \( \beta \): replacement rate of SRU; \( N_i \): total number of LRU on the radar equipment; \( k_i \): minimum amount of LRU required for normal operation of the system; \( S_{LRU_i} \): grass-root level inventory of LRU; \( q_y \): weight of SRU; \( S_{SRU_{ij}} \): amount of SRU at the depot-level warehouse; \( S_{CSRU_{ij}} \): amount of SRU at the grass-root level.
warehouse; \(T_r\) : normal running time of radar equipment in one period; \(T_{sp}\) : replacement time of radar equipment in one period; \(r_i\) : number of staff for replacement; \(\varepsilon_i\) : maintenance rate for replacement; \(p_{x_i}\) : probability of \(x_i\) that will be repaired or queued for maintenance.

**Operation Cycle of Radar Equipment**

This paper defines the operation cycle of phased-array radar equipment from beginning to work until the next normal operation, namely the amount of the failure LRU is from zero to next zero, as shown in figure 3.

![Figure 3. Work cycle of phased array radar equipment.](image)

**Expected Fill Rate of Spare Parts**

Phased array radar equipment has a large number of LRU, a single failure LRU does not affect the performance of antenna, thus phased-array radar equipment can be regarded as a typical k/N system [6-9], types of LRU in the system are consistent, failure rate of SRU can be used to express the failure rate of LRU, and the weight of each SRU can be expressed as

\[
\begin{align*}
\lambda_i &= \sum_{j=1}^{m} \lambda_j, \\
\varphi_i &= \frac{\lambda_j}{\lambda_i}
\end{align*}
\]

\(\alpha\) is defined as the feasibility of cannibalization, that is the percentage of LRU that are repaired with cannibalization, and the number of LRU that is fixed with cannibalization can be expressed as \(m_{\alpha} = \lceil \alpha m \rceil\), \(\beta\) is defined as the replacement rate of SRU, that is the percentage of LRU that are repaired with replacing SRU, thus the number of LRU that are repaired with replacing SRU can be expressed as \(m_{\beta} = \lceil \beta m \rceil\), the number of LRU that are replaced can be expressed as \(m_{LRU} = S_{LRU} = \lceil (1 - \alpha - \beta)m \rceil\).

According to the characteristics of k/N system, when failure number of LRU; reaches \(m\), the reliability of antenna can be expressed as

\[
R(t) = \sum_{j=1}^{m} C_{N_j}^j [r_j(t)]^{N_j-1} [1-r_j(t)]^j = \sum_{j=1}^{m} C_{N_j}^j [\exp(-\lambda_j t)]^{N_j-1} [1 - \exp(-\lambda_j t)]^j
\]

Normal working hours of LRU; are expected to be as [10]

\[
E(T) = \int_0^\infty R(t) \, dt = \sum_{j=1}^{m} \frac{1}{(N_j-\alpha)\lambda_j}
\]

Phased array radar equipment is composed of several series of k/N systems, because the failure rate of T/R modules is much higher than power units and composition units, when the failure number of T/R modules reaches the threshold, the failure number of power units and composition units are far from reaching the threshold, so this article selects expected time that when the number of failure T/R modules reaches the threshold as the working hours of system, and then uses the
formula (4) to calculate the failure number of power units and composition units when the equipment stop to be repaired.

\[
\sum_{x=0}^{N_i-1} \frac{1}{(N_i-a)\lambda_i} = \sum_{x=0}^{N_i-1} \frac{1}{(N_i-a)\lambda_2} = \sum_{x=0}^{N_i-1} \frac{1}{(N_i-a)\lambda_3}
\]  

(4)

In normal working time, for the malfunction’s randomicity, the probability distribution function of LRU and SRU can be expressed as

\[
\begin{align*}
p(x_i = j | T = T_i) &= C_{i}'(e^{\lambda_i T_i})^{-j}(1-e^{\lambda_i T_i})^{j-1} \\
p(x_s = j | T = T_s) &= C_{s} e^{\lambda_s T_s}(1-e^{-\lambda_s T_s})^{j-1}
\end{align*}
\]

(5)

Because the depot-level repair workshop does not need to repair LRU, so the expected fill rate of LRU can be expressed as

\[
EFR(s_{LRU}) = \sum_{j=0}^{s_{LRU}-1} p(x_i = j | T = T_i)
\]

(6)

Because different types of LRU are series structure, the overall expected fill rate of LRU can be expressed as

\[
EFR(s_{LRU}) = \prod_{i=1}^{J} EFR(s_{LRU})
\]

(7)

After cannibalization and replacement of SRU, the failure SRU is sent to the depot-level repair workshop. This paper assumes that there are \( c_i \) servicing equipment, the repair process of failure SRU can be regarded as a queuing system which has \( m_y = m \) customers and \( c_i \) service counter, whose arrival rate is \( \lambda_i \) and whose repair rate is \( \mu_i \). Among them, arrival rate and replacement time can be expressed

\[
\begin{align*}
\lambda_i &= \frac{1}{T_i + T_{gr}} \\
T_{gr} &= \frac{m_y}{r_i c_i}
\end{align*}
\]

(8)

In the analysis, it is assumed that the queue customer’s length is smaller than \( 2m_y \). According to the queuing theory, state transfer flow of the customer can be drawn in figure 4.

\[
\rho_i = m_i \frac{\lambda_i}{c_i \mu_i}
\]

is defined as service intensity of the customer’s queuing system, and when its value is less than 1, the queuing system can finally achieve the stable equilibrium, and the transition probability of the above customers can be expressed as

\[
\begin{align*}
p_{x_i,0} &= (\lambda_i + x_i \mu_i) = p_{x_i,0} \cdot (x_i + 1) \mu_i, \quad 0 \leq x_i < c_i \\
p_{x_i,c_i} &= (\lambda_i + c_i \mu_i) = p_{x_i,c_i} \cdot c_i \mu_i, \quad c_i < x_i < m_y \\
p_{x_i,m_y} &= p_{x_i,m_y} \cdot \lambda_i + p_{x_i,m_y} \cdot c_i \mu_i, \quad m_y < x_i < 2m_y
\end{align*}
\]

(9)

The scale factor \( d_i = \lambda_i / \mu_i \), formula (9) can be simplified
\[
\begin{align*}
    p_{x_y} &= (d_x + x_y) = p_{x_y+l}(x_y + 1), \quad 0 \leq x_y \leq c_y \\
    p_{x_y} &= (d_x + c_y) = p_{x_y+l} c_y, \quad c_y < x_y \leq m_y \\
    p_{x_y} &= c_y = p_{x_y+l} d_x + p_{x_y+l} c_y, \quad m_y < x_y < 2m_y
\end{align*}
\]

Assume that

\[
\begin{align*}
    \alpha_{x_y} &= \prod_{j=0}^{x_y-1} \frac{(d_x + j)}{x_y!} \\
    \beta_{x_y} &= (\frac{d_x + c_y}{c_y})^{x_y-c_y} \\
    \gamma_{x_y} &= \sum_{j=0}^{x_y-c_y-1} \frac{p_x d_j}{c_y}
\end{align*}
\]

The formula (10) can be further simplified

\[
\begin{align*}
    p_{x_y} &= \begin{cases} 
    p_{b_0} x_y = 0 \\
    p_{b_0} \alpha_{x_y}, 0 < x_y \leq c_y \\
    p_{b_0} \alpha_{x_y} \beta_{x_y}, c_y < x_y \leq m_y \\
    p_{b_0} \alpha_{x_y} \beta_{x_y} - \gamma_{x_y}, m_y < x_y < 2m_y
    \end{cases}
\end{align*}
\]

Because the sum of all the reliable probabilities is equal to 1

\[
p_{b_0} = 1 + \sum_{x_y=1}^{c_y} \alpha_{x_y} + \sum_{x_y=m_y+1}^{x_y=m_y+1} \alpha_{x_y} \beta_{x_y} + \sum_{x_y=m_y+1}^{x_y=x_y+1} (\alpha_{x_y} \beta_{x_y} - \sum_{j=0}^{x_y-m_y-1} \frac{p_x d_j}{c_y}) = 1
\]

When solving the expected fill rate of SRU_{ij}, two conditions need to be satisfied: 1. The demand of SRU \( x_y \leq S_{CSRU_{ij}} \); 2. The number of maintenance or queue for maintenance is less than or equal to \( S_{CSRU_{ij}} \), so the expected fill rate of SRU_{ij} can be expressed as

\[
EFR(S_{SRU_{ij}}) = \sum_{j=0}^{S_{CSRU_{ij}-1}} p(x_y = j | T = T_y) \sum_{x_y=0}^{S_{CSRU_{ij}-1}} P_{x_y}
\]

**Spare Parts Configuration Model and Solution Algorithm**

**Spare Parts Configuration Model**

According to the characteristics of two-level maintenance system of phased array radar equipment, the authors defined Q as the total cost of Spare parts configuration, the optimization model of the system can be briefly described as minimizing the cost of total spare parts configuration under the condition of specified spare fill rate.

Object function: \( \min Q = \sum_{i=1}^{S_{LRU}} q_{LRU} + \sum_{i=1}^{S_{CSRU}} q_{CSRU} + \sum_{i=1}^{S_{SRU}} q_{SRU} \)

\[
\begin{align*}
    \text{Constraint condition:} \quad & \begin{cases} 
    EFR(S_{LRU}) \geq A_L \\
    EFR(S_{SRU}) \geq A_S
    \end{cases}
\end{align*}
\]
Solution Algorithm

Some foreign software such as VMETRIC and OPUS, the marginal algorithm is preferred when they are used to calculate the optimizing inventory of spare parts, the traditional marginal algorithm is to compare the marginal benefits generated when the spare parts are increased by 1, then add 1 to the spare parts according to whose marginal benefits is the largest, and the quantity of other spare parts remains unchanged until reaching the expected fill rate. This method needs to start iterating from 0 and the optimization efficiency is low. In order to improve the efficiency, the paper firstly decomposes the expected fill rate into LRU and SRU, determines the quantitative inventory range of each spare part, and uses the marginal benefit method for iterative calculation, finally determines the configuration of LRU and SRU, and the flow chart of solution algorithm is shown in Figure 5.

![Flow chart of solution algorithm.](image)

Figure 5. Flow chart of solution algorithm.

Example Simulation Analysis

Assuming that a large phased array radar is mainly composed of T/R modules (LRU\_1), power units (LRU\_2), and the composition units (LRUs), each LRU is composed of several kinds of SRU in series, and the three units are redundant structures, which can be regarded as three k/N systems of 1000/1200, 570/600, 280/300, the minimum expected fill rate of LRU \( A_1 = 0.8 \), the minimum expected fill rate of SRU \( A_2 = 0.97 \), the specific parameters of LRU and SRU are shown in table 1 and 2.

<table>
<thead>
<tr>
<th>Table 1. Concrete parameters of each LRU.</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter</td>
</tr>
<tr>
<td>( \lambda_i \times 10^{-4} )</td>
</tr>
<tr>
<td>( q_i / 10^4 )</td>
</tr>
<tr>
<td>( r_i )</td>
</tr>
<tr>
<td>( \varepsilon_i \times 10^{-1} )</td>
</tr>
</tbody>
</table>
Table 2. Concrete parameters of each SRU.

<table>
<thead>
<tr>
<th>parameter</th>
<th>$\lambda_i/10^{-3}$</th>
<th>$q_i/10^4$</th>
<th>$c_i$</th>
<th>$\mu_i/(h^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SRU_{11}$</td>
<td>2</td>
<td>0.6</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>$SRU_{12}$</td>
<td>2</td>
<td>0.5</td>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td>$SRU_{13}$</td>
<td>1</td>
<td>0.4</td>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td>$SRU_{21}$</td>
<td>0.5</td>
<td>0.2</td>
<td>3</td>
<td>0.05</td>
</tr>
<tr>
<td>$SRU_{22}$</td>
<td>0.5</td>
<td>0.3</td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>$SRU_{31}$</td>
<td>0.3</td>
<td>0.2</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>$SRU_{32}$</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
<td>0.05</td>
</tr>
</tbody>
</table>

According to the formula (4), it can be calculated that when the failure number of T/R modules is 200, the failure number of power units is 20, and the failure number of composition units is 5. The concrete steps are follows:

Step 1 According to the minimum expected fill rate of LRU $A_t = 0.8$, and the series structure between three kinds of LRU, the minimum expected fill rate assigned to each item of LRU is 0.8, according to formula (6) , the initial stock number of three LRU are 129, 15 and 5.

Step 2 After determining the number of initial configurations, the marginal efficiency of the three LRU is calculated, then add 1 to the spare parts according to whose marginal benefits is the largest. After several iterations, the final stock number of three LRU is 130, 19, 8.

Step 3 Selecting the appropriate parameters $\alpha$ $\beta$, in order to illustrate the problem, this paper selects parameters $\alpha=20\%$, $\beta=15\%$, by using the improved marginal efficiency algorithm designed in this paper, the author calculates the inventory of each SUR at grass-root level warehouse and depot-level warehouse, the results are shown in table 3.

Table 3. Inventory of each SUR at grass-root level and depot-level.

<table>
<thead>
<tr>
<th>parameter</th>
<th>Inventory of grass-root level</th>
<th>Inventory of depot-level</th>
<th>Assignment cost</th>
<th>Expected fill rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SRU_{11}$</td>
<td>7</td>
<td>19</td>
<td>35.8</td>
<td>0.970326</td>
</tr>
<tr>
<td>$SRU_{12}$</td>
<td>7</td>
<td>25</td>
<td>35.8</td>
<td>0.970068</td>
</tr>
<tr>
<td>$SRU_{13}$</td>
<td>4</td>
<td>7</td>
<td>35.8</td>
<td>0.97653</td>
</tr>
<tr>
<td>$SRU_{21}$</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>0.9775</td>
</tr>
<tr>
<td>$SRU_{22}$</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>0.9719</td>
</tr>
<tr>
<td>$SRU_{31}$</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0.9871</td>
</tr>
<tr>
<td>$SRU_{32}$</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0.9894</td>
</tr>
</tbody>
</table>

The total cost of the system is the sum of the cost of LRU configuration and the cost of the SRU configuration, that is $Q = 35.8 + 3 + 1.6 + 130 \times 10 + 19 \times 4 + 8 \times 2 = 1.4324 \times 10^7$, in order to further illustrate the effect of parameters $\alpha$ $\beta$ on the inventory decision, the author study on the relationship between the cost of SRU which is belong to T/R module and expected fill rate under different parameters $\alpha$ $\beta$, the results are shown in Figure 6.

Figure 6. Relationship between the cost of SRU which is belong to T/R module and expected fill rate.
According to the calculation result of Step 2, when the stock number of T/R module is 130, we can obtain that $\alpha + \beta = 35\%$, figure 6 can clearly show the relationship between the cost of SRU (belong to T/R module) and expected fill rate under 4 kinds of $\alpha$ $\beta$ parameters. Under certain condition of expected fill rate, the results show that the higher proportion of $\alpha$ parameters (the rate of LRU repaired by Cannibalization), the lower assignment cost of SRU.

**Conclusion**

This paper selects three kinds of LRU and the SRU which is subordinate to the LRU as the research object, establishes a spares of LRU and SRU configuration model based on cannibalization under the two-level maintenance, then the paper further analyzes the relationship between the cost of SRU (belong to T/R module) and expected fill rate, the result show that the higher $\alpha$ parameter or the lower $\beta$ parameter, the lower assignment cost of SRU, Which will have strong military and economic significance.

**References**


