Performance Comparison of Different Dispersion Compensation Schemes in Quasi-linear Transmission System with Intrachannel Four-wave Mixing

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Abstract. Intrachannel four-wave mixing (IFWM) is the dominate source of bit errors in high-bit-rate quasi-linear fiber-optic transmission systems. Previous studies have shown that IFWM can be reduced by proper dispersion compensation scheme. In this paper, different schemes like pre-, post-, and symmetrical-dispersion compensations are compared numerically in the presence of IFWM in return-to-zero quasi-linear transmission systems. The performances of the three compensation schemes have been compared in terms of the average intensity deviation of the “1” bits and the relative intensity of the ghost pulse (the “0” bit). It is found that, for a fixed amplifier spacing, optimal compensation scheme (with minimum average intensity deviation of the “1” bits and minimum intensity of the ghost pulse) depends on transmission bit rate. As bit rate increases the performance of pre-compensation becomes better and better. On the contrary, post-compensation exhibits its advantage more and more as bit rate decreases.

Introduction

In high-bit-rate long-haul quasi-linear fiber-optic transmission systems (at 40 Gb/s per channel and above), intrachannel nonlinear effects such as intrachannel four-wave mixing (IFWM) and intrachannel cross-phase modulation (IXPM) are the main sources of bit errors [1]. While IXPM can be effectively suppressed by strong dispersion management in combination with a return-to-zero (RZ) modulation format, IFWM remains a limiting factor which results in amplitude jitter (or intensity deviation) in the “1” bits and the generation of ghost pulses through energy transfer from the “1” bits to the “0” bits in on–off keying (OOK) systems [2-4]. It has been shown that, due to the interplay between chromatic dispersion and intrachannel nonlinear effects, system performance depends on the position of dispersion compensated fibers [5] and that intrachannel nonlinearities may be partially reduced by proper dispersion map design [6].

There are three basic dispersion compensation schemes, pre-, post-, and symmetrical compensations, and performance comparison of transmission with these schemes has been carried out [7-12]. However, most of the earlier studies [7-10] didn't consider or discuss the IFWM effect because the single channel rate considered there was limited to 10 Gb/s where IFWM was of less importance than other nonlinear effects such as self-phase modulation (SPM) and cross-phase modulation (XPM). It is known that IFWM is caused by the interplay of different frequency components of ultrashort pulses within the same channel. The higher the channel rate is the narrower the RZ pulses should be, and the dispersive nature of the pulses gives rise to large pulse overlap and thus large IFWM effect. Randhawa et al. [11] investigated the three compensation schemes in the presence of fiber nonlinearities in 40 Gb/s carrier-suppressed RZ systems and found that symmetrical compensation is the best one which reduces the bit error rate (BER) to the more extent than that of pre- and post-compensations. They also showed that with increase in the input bit rate, symmetrical compensation was still the best, i.e., the optimal compensation scheme (with minimum BER) was independent of transmission bit rate. Note that the IFWM effect was not discussed in Ref.[11].

Recently, performance comparison of the three compensation schemes for different fiber standards has been carried out by numerical simulations [12]. It is observed that, for a fixed bit rate,
the choice of compensation schemes depends on the characteristics of the transmission fiber. For example, pre-compensation is the best for ITU 655 fiber while Alcatel fiber is best for post- and symmetrical compensation. However, the simulation only focused on a single bit rate of 40 Gb/s, and the effect of IFWM on the transmission performance was not discussed.

In this paper, pre-, post-, and symmetrical compensations are compared numerically in the presence of IFWM for 80, 40 and 20 Gb/s RZ quasi-linear transmission systems. The simulations are based on a set of four coupled nonlinear Schrödinger equations which include intrachannel nonlinearities such as IFWM, IXPM and SPM. It is found that the optimal compensation scheme depends on the transmission bit rate. Pre-compensation is the best for high bit rate such as 80 Gb/s, whereas post-compensation becomes the best for low bit rates such as 40 and 20 Gb/s.

**Simulation Setup**

Numerical simulation was performed for the transmission links schematically shown in Fig. 1, where (a), (b) and (c) represent, respectively, pre-, post- and symmetrical compensations. Signals from the optical transmitter enter the periodic transmission links. The period length is the same for all three compensation schemes. Each period comprises a 72.77 km long standard single-mode fiber (SSMF), 7.277 km long dispersion-compensating fiber (DCF), and an EDFA. Note that in the symmetrical compensation scheme the 7.277 km long DCF is divided into two segments connecting to both ends of the SSMF. We assume that the DCF can exactly compensate for chromatic dispersion of the SSMF so that the average dispersion of the transmission link is zero. The EDFA exactly compensates for energy loss caused by the SSMF and DCF.

![Simulation Setup](image)

**Basic Equations**

The simulation is based on the generalized nonlinear Schrödinger equation which takes the form

\[
\frac{i}{\xi} \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + \mu \left| u \right|^2 u = -\frac{i}{2} \Gamma u + \frac{i}{2} \mu u ,
\]

where \(\xi, \tau,\) and \(u\) are, respectively, the normalized distance, time, and field envelope in soliton units. The parameters \(\Gamma\) and \(\mu\) account for, respectively, the fiber loss and the gain of the EDFA.
The second term on left side represents group-velocity dispersion (GVD) where the sign “+” or “−” is chosen, respectively, when the field is transmitted in the SSMF (anomalous GVD) and the DCF (normal GVD). The third term on left side represents the Kerr nonlinearity.

Although a pseudo-random bit stream as an input is essential for accurate description of signal transmission in a realistic system, it needs tediously programming. However, considerable physical insight could be gained with a limited number of input bits[3,13,14] when we only focus our attention on the relative comparison of the three compensation schemes. Here the input is assumed to be the sum of four bits in the form

$$u(0, \tau) = u_1(0, \tau + 3q_0) + u_2(0, \tau + q_0) + u_3(0, \tau - q_0) + u_4(0, \tau - 3q_0)$$

$$= A_i \text{sech}(0, \tau + 3q_0) + A_i \text{sech}(0, \tau + q_0) + A_i \text{sech}(0, \tau - q_0) + A_i \text{sech}(0, \tau - 3q_0). \quad (2)$$

Where $2q_0$ represents the duration of the bit slot and $A_i (j=1,2,3,4)$ represents the amplitude of the $j$th bit. We assume that all the “1” bits have the same initial width and the same amplitude and that the “0” bits have a much smaller amplitude than that of the “1” bits. Substituting the input into Eq. 1, we obtain a set of four coupled equations:

$$i \frac{\partial u_1}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_1}{\partial \tau^2} + \frac{i}{2} \Gamma u_1 - \frac{i}{2} \mu u_1$$

$$= -\left(u_1^2 + 2u_2^2 + 2u_3^2 + 2u_4^2\right)u_1 - u_2^2u_3^* - 2u_2u_4u_3^*,$$  

$$i \frac{\partial u_2}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_2}{\partial \tau^2} + \frac{i}{2} \Gamma u_2 - \frac{i}{2} \mu u_2$$

$$= -\left(u_2^2 + 2u_1^2 + 2u_3^2 + 2u_4^2\right)u_2 - u_1^2u_3^* - 2u_1u_4u_3^* - 2u_2u_4u_3^*$$,  

$$i \frac{\partial u_3}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_3}{\partial \tau^2} + \frac{i}{2} \Gamma u_3 - \frac{i}{2} \mu u_3$$

$$= -\left(u_3^2 + 2u_1^2 + 2u_2^2 + 2u_4^2\right)u_3 - u_1^2u_2^* - 2u_1u_2u_3^* - 2u_2u_4u_3^*$$,  

$$i \frac{\partial u_4}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_4}{\partial \tau^2} + \frac{i}{2} \Gamma u_4 - \frac{i}{2} \mu u_4$$

$$= -\left(u_4^2 + 2u_1^2 + 2u_2^2 + 2u_3^2\right)u_4 - u_1^2u_2^* - 2u_1u_2u_3^*.$$  

In real parameters

$$\xi = \frac{z}{L_D} = \frac{2|\beta_0|}{T_0^2}, \quad \tau = \frac{t - z}{v_g T_0}, \quad \Gamma = \alpha L_D = \frac{\alpha T_0^2}{|\beta_0|}, \quad \mu = (\gamma_0 - \alpha)L_D.$$

Where $z$, $t$, $v_g$ represent, respectively, distance, time, and group velocity. $T_0$ is the half-width (at 1/e-intensity point) of the input “1” bits, $\beta_0$ is the GVD coefficient, $\alpha$ is the attenuation constant of the fiber, $\gamma_0$ is the unsaturated gain parameter of the EDFA, and $L_D = T_0^2/|\beta_0|$ is the dispersion length. We do not include the Raman self-scattering (RSS) and self-steepening effects because, for quasi-linear strongly dispersion-managed transmission, the path-averaged bit width is very large and the peak power is very low. We also neglect the amplifier noise and believe that it will has a small influence on the relative comparison of the three compensation schemes. Eq. 3-6 and their modified version can be used to describe bits transmission in the SSMF, DCF and EDFA. The differences are that, for transmission in the SSMF and the DCF, the parameter $\mu$ is zero, while for transmission in the EDFA, the loss term ($\Gamma$) and even all the nonlinear terms can be neglected. In real parameters, the relation between the initial amplitude $A_j$ of the $j$th bit in Eq. 2 and its real peak power is given by
and the energy of a hyperbolic-secant pulse with peak power $P_j$ and pulse width $T_0$ is

$$E_{\text{sech}} = 2P_j T_0,$$

where $\gamma$ is the nonlinearity coefficient of the fiber.

## Simulation Results and Discussion

### The Effect of IFWM on Signal Transmission

Quasi-linear transmission operates in the regime in which the local dispersion length $L_D$ is much shorter than the nonlinear length $L_{NL}$ in all fiber sections of a dispersion-managed link [1], where

$$L_{NL} = \frac{1}{\gamma P_j},$$

(10)

According to Eq. 8, the quasi-linear condition requires that

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_j^2}{|\beta_2|} = A_j^2 \ll 1.$$

(11)

A typical example of quasi-linear transmission is a system operating at bit rates of 40 Gb/s or more and employing ultra-short pulses that spread quickly over multiple bits as they propagate along the link, which is just the case discussed throughout this paper. It is shown that IFWM is the most important intrachannel nonlinear effect in quasi-linear dispersion-managed transmission system [14]. So, before we compare the three compensation schemes, it is useful to see how IFWM affects signal transmission.

It is shown that [14] the influence of IFWM on signal transmission depends on bit patterns. For 4-bit inputs, the patterns like 1110 and 0110 which contain consecutive “1” bits manifest large distortion both in shape and spectrum, whereas the distortion is very small in the case of non-consecutive “1” bits such as 1010 and 1001. Therefore, we focus our attention on the transmission of bit pattern 1110. In all cases, the transmission link is fixed and is the same as described in Section 2, i.e., each transmission period comprises a 72.77 km long SSMF, 7.277 km long DCF, and an EDFA which exactly compensates for the energy loss. The GVD coefficients of the SSMF and DCF are assumed to be, respectively, $(\beta_2)_{\text{SSMF}}=-20 \text{ ps}^2/\text{km}$ and $(\beta_2)_{\text{DCF}}=200 \text{ ps}^2/\text{km}$ near 1.55 $\mu$m. The DCF is assumed to has the same attenuation constant and the same nonlinearity coefficient as those of the SSMF with $\alpha=0.046 \text{ km}^{-1}$ and $\gamma=1.3 \text{ W}^{-1}\text{km}^{-1}$. Actually, since the DCF is much shorter than the SSMF, neglecting the difference of the parameter $\alpha$ and $\gamma$ between the SSMF and the DCF will have a negligible influence on the simulation results.

First, we consider transmission with all the mentioned nonlinear and dispersion effects except IFWM. Fig. 2 shows the simulation results (outputs) of the three compensation schemes in both (a) shape and (b) spectrum for bit pattern 1110, where the green dotted curve represents the input, the blue solid, black dashed-dotted, and red dashed curves represent, respectively, the outputs of pre-, post-, and symmetrical-compensations. In all cases the transmission distance is fixed at 960 km that is equal to 12 transmission periods as shown by Fig.1. The input is assumed to be in the form of Eq. 2, where all the “1” bits have a same initial width of $T_{\text{FWHM}}=3 \text{ ps}$ ($T_0=T_{\text{FWHM}}/1.763\approx1.7 \text{ ps}$) and a same initial amplitude of $A_j=0.2$ ($j=1,2,3$) which, according to Eq. 8, corresponds to a peak power of 213 mW, satisficing the quasi-linear condition. The “0” bit has the same initial width and shape as those of the “1” bits but with much smaller peak power of $2.13 \text{ mW}$ ($A_4=0.02$). The initial separation ($2q_0\times T_0$) between two adjacent bits is 12.5 ps, representing a bit rate of 80 Gb/s. The intensities of the pulse shape and spectrum are normalized with respect to the input, respectively.
We see that, in the absence of IFWM, the effects of GVD, SPM and IXPM have a small influence on transmission in both shape and spectrum, and the difference among the three compensation schemes are very small. However, the situation is different when IFWM is added into consideration which is shown by Fig. 3. Obviously IFWM is the dominant nonlinear effect for a typical dispersion compensated quasi-linear transmission system. Because of dispersion-broadened pulse overlapping, time-domain four-wave mixing transfers energy from triples of pump pulses into the “0” bit, generating ghost pulse, and into the “1” bits resulting in intensity deviation. Moreover, IFWM exerts different influence on different compensation schemes. In this case the pre-compensation seems to be the best because of the lowest intensity fluctuation of the “1” bits and the smallest energy of the ghost pulse. On the contrary, the post-compensation gives a worst result since the intensities of the “1” bits fluctuate dramatically and the energy of the ghost pulse is even larger than that of one of the “1” bits.

In the next subsection performance comparison between different compensation schemes will be made in more detail at transmission bit rates of 80, 40, and 20 Gb/s, respectively.

**Performance Comparison of the Three Compensation Schemes**

In this subsection we compare the performance of pre-, post- and symmetrical compensations at transmission bit rates of 80, 40, and 20 Gb/s, respectively. The hyperbolic-secant pulse widths $T_{FWHM}$ are, respectively, 3, 6, and 12 ps, thus each bit rate has the same duty cycle. In all cases, the transmission link is the same as that for Figs. 2 and 3, and the input bit pattern is 1110.
To make the comparison quantitative, we calculate two parameters, i.e., the average peak intensity deviation ($\Delta P_{\text{aver}}$) of the “1” bits and the relative peak intensity of the ghost pulse generated at the “0” bit slot. The relative peak intensity of the ghost pulse is defined as the ratio of the peak intensity of output ghost pulse to the peak intensity of the input “0” bit (note that, as mentioned earlier, the input “0” bit is assumed to have a very small amplitude). The average peak intensity deviation ($\Delta P_{\text{aver}}$) of the “1” bits is given by

$$\Delta P_{\text{aver}} = \frac{\sum_{j=1}^{3} |P_{j(\text{out})}-1|}{3},$$

where $P_{j(\text{out})}$ is the peak intensity of the $j$th output “1” bit. Note that, as shown by Figs. 2 and 3, all the intensities are normalized with respect to the peak intensity of the input “1” bit which has a normalized value of 1.

Fig. 4 compares the results of the three compensation schemes, where (a) and (b) show, respectively, the variation of $\Delta P_{\text{aver}}$ and the relative peak intensity of the ghost pulse with transmission distance. In all cases, the transmission distance is fixed at 960 km, the input is the same as that for Fig. 3, i.e., all the “1” bits have a same initial width of $T_{\text{FWHM}}=3$ ps and a same initial peak power of 213 mW. The separation between two adjacent bits is 12.5 ps representing a bit rate of 80 Gb/s. Fig. 4(a) shows that for all compensation schemes the value of $\Delta P_{\text{aver}}$ increases with transmission distance. Pre-compensation results in a smallest value of $\Delta P_{\text{aver}}$ while post-compensation causes the largest value of $\Delta P_{\text{aver}}$. Furthermore, as shown by Fig. 4(b), post-compensation generates the largest ghost pulse, whereas pre- and symmetrical-compensations are much better for suppression of the ghost pulse. In general, pre-compensation is the best and post-compensation is the worst in this case.

Fig. 5 gives a similar comparison at a bit rate of 40 Gb/s, where all the “1” bits have a same initial width of $T_{\text{FWHM}}=6$ ps. The separation between two adjacent bits is twice as big as that for Fig. 4, so the duty cycle is the same as that for Fig. 4. The input peak power of the “1” bit is half of that for Figs. 2-4, thus, according to Eq. 9, the “1” bit contains the same energy as that for Figs. 2-4.

![Figure 4. Comparison of pre-, post-, and symmetrical-compensations at 80 Gb/s for bit pattern 1110. Variation of (a) the average peak intensity deviation of the “1” bits ($\Delta P_{\text{aver}}$) and (b) the relative peak intensity of the ghost pulse with transmission distance.](image-url)
Fig. 5(a) shows that, relative to the case of Fig. 4, the value $\Delta P_{\text{aver}}$ of pre-compensation is increased while $\Delta P_{\text{aver}}$ of post-compensation decreased, and the same feature applies to the ghost pulse as shown by Fig. 5(b). Relatively, the variation of the symmetrical compensation is less obvious. This means that the relative performance of the three compensation schemes changes with transmission bit rate. The reason is that different bit rates correspond to different initial pulse widths which leads to different degree of pulse overlapping because of GVD, the IFWM effect depends on pulse overlapping and causes the performance difference of the compensation schemes.

To further prove it, we further reduce the bit rate to 20 Gb/s, and the results are shown by Fig. 6. Where the “1” bits have an initial width of $T_{\text{FWHM}}=12$ ps, the duty cycle and the pulse energy are identical to those for Figs. 2-5. It is seen that the performance of post-compensation is further improved while pre-compensation becomes the worst one, which is exactly the opposite of the case of Fig. 4. The conclusion here is that pre-compensation is the best for high bit rates such as 80 Gb/s, while post-compensation is suitable for low bit rates such as 40 and 20 Gb/s. The overall trend is that as bit rate decreases the performance of post-compensation becomes better and better, on the contrary, pre-compensation exhibits its advantage more and more as bit rate increases.

The research thus far has been concentrated on different bit rates with equal duty cycle. We also compared the three compensation schemes at bit rates of 80, 60, 40 and 20 Gb/s, respectively, but with unequal duty cycles. In all cases, the transmission link and the bit pattern 1110 are identical to those used for Figs. 2-6. The hyperbolic-secant pulses representing the “1” bits have the same energy and same width of $T_{\text{FWHM}}=3$ ps for all bit rates and for all compensation schemes, so the
duty cycles are different for different bit rates, i.e., 0.24, 0.18, 0.12, and 0.06, respectively, for 80, 60, 40 and 20 Gb/s. Simulations not shown here revealed that the main feature was similar to that of Figs. 4-6, and the previous conclusions are also applicable for this case. The reason is easy to understand. Since the pulse width and pulse energy are identical for all bit rates, different duty cycles lead to different degree of pulse overlapping, IFWM causes performance discrepancies of the three compensation schemes.

It should be pointed out that all previous discussions are based on the quasi-linear condition described by Eq. 11 with $A_j=0.2$–$0.4$ where $A_j$ is the amplitude of the “1” bit. Indeed, further simulations indicated that the above conclusions are justified as long as $A_j<0.5$. However, the situation will be quite different when $A_j$ exceeds 0.5, as the soliton effect plays an important role in this case, which is beyond the scope of our discussion.

**Summary**

We have compared the performances of pre-, post-, and symmetrical-dispersion compensations in the presence of IFWM in 80-20 Gb/s RZ quasi-linear transmission systems. Numerical simulations show that, for a fixed amplifier spacing, the optimal compensation scheme depends on transmission bit rate. As bit rate increases the performance of pre-compensation becomes better and better. On the contrary, post-compensation exhibits its advantage more and more as bit rate decreases. Thus, pre-compensation is the best candidate for high bit rates and post-compensation is suitable for low bit rates.

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**References**


