A New DOA Estimation Method Based on Spatial Structure of Array Signals

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ABSTRACT

DOA Estimation is a core item of smart antenna in FDD system. In this paper, an equidistant linear array is divided into two identical sub-arrays, and then a DOA matrix is constructed, the eigenvector matrix of which is the array manifold of the signal. After obtaining the array manifold through eigenvalue decomposition, the DOA of signal can be estimated further. Basing on the theoretical analysis, computer simulation is presented in this paper.

KEYWORDS

FDD; DOA; Signal Spatial Structure

INTRODUCTION

Along with the development of society and economy, mobile communication has become one of the main methods of communication. The rapid development of the market requires more and more advanced technologies to ensure the capacity and quality of communication systems. In recent years, smart antenna has been receiving increasing attention, which can effectively improve capacity of system and better system’s signal-noise ratio. Smart antenna can improve the frequency utilization rate and further improve system’s capacity when it is used in TDMA system. Meanwhile, smart antenna can also be combined with different duplex
modes. In the TDD mode, since the identical frequency is used by up-link and
down-link signals, the directional emission of the down-link beam can use the
parameters obtained by the up-link beam-forming. In contrast, the frequencies used
by up-link and down-link are different in the FDD mode, the directional emission of
the down-link beam can’t use the parameters obtained by the up-link beam-forming.
Under this circumstance, the direction-of-arrival (DOA) of up-link is the only
parameter can be utilized and is also the only bridge connecting the up-link and
down-link signals. Therefore, the DOA estimation is a core question of smart
antenna technology in the FDD mode[1].

In document [2], two parallel linear antenna arrays are used to evaluate the two-
dimensional arrival angle of radar signals. In mobile communication, due to the
distance between base station and mobile station far outweigh the height of base
station antenna, the azimuth of the arrival signal is generally only considered
irrespective of the elevation angle. Under the document[2], a linear array is divided
into two sub-arrays of equal structure to construct the DOA matrix of the signal, and
then determining the arrival angle by the eigenvalue decomposition.

**SIGNAL MODE**

Supposing we use a linear array, and the number of element is \( M_0 \). Dividing the
antenna array into two sub-arrays, the array element number of the first sub-array is
\((1,2,\ldots,M)\) , and the other is \((L, L+1,\ldots,L+M-1)\) , \( M_0 = M + L - 1 \). Where,
\( L >= 2 \) , is an integer.

Assuming the total number of users is \( K \ (K < M) \), the signal received by the
array element \( k \) in the first sub-array can be expressed as:

\[
x_k(t) = \sum_{i=1}^{K} e^{j(k-1)\frac{\omega d}{c}\sin \theta_i} s_i(t) + n_k(t) \quad k = 1,2,\ldots,M \quad (1)
\]

The signal received by the array element \( k \) in the second sub-array can be
expressed as:

\[
y_k(t) = \sum_{i=1}^{K} e^{j(k+L-2)\frac{\omega d}{c}\sin \theta_i} s_i(t) + n_{k+L-1}(t) \quad k = 1,2,\ldots,M \quad (2)
\]

Where, \( s_i(t) \) is the modulation signal of the user \( i \) , \( n_k \) is the noise received by
the array element \( k \) , \( \omega \) is the carrier frequency.

Using the matrix representation, we can re-express the \( x_k(t) \) and \( y_k(t) \) as:

\[
X(t) = AS(t) + N_x(t) \quad (3)
\]
\[ Y(t) = A\Phi S(t) + N_y(t) \quad (4) \]

where, \[ X(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T, \]
\[ Y(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T, \]
\[ N_x(t) = [n_1(t), n_2(t), \ldots, n_M(t)]^T, \]
\[ N_y(t) = [n_{L}(t), n_{L+1}(t), \ldots, n_{L+M-1}(t)]^T, \]
\[ A = [a_1(\theta_1), a_2(\theta_2), \ldots, a_K(\theta_K)], \]
\[ \Phi = \text{diag} [e^{j\frac{d}{c}(L-1)\sin(\theta_1)}, e^{j\frac{d}{c}(L-1)\sin(\theta_2)}, \ldots, e^{j\frac{d}{c}(L-1)\sin(\theta_K)}] \]
\[ a_i(\theta_i) = [1, e^{j\frac{d}{c} \sin(\theta_i)}, \ldots, e^{j(M-1)\frac{d}{c} \sin(\theta_i)}]^T, \quad i = 1, 2, \ldots, K. \]

Aiming at this signal mode, we can make supposing like following[4]: \( M > K \), and \( a_1(\theta_1), a_2(\theta_2), \ldots, a_K(\theta_K) \) are linear independent of each other. The noise is a Gaussian random variable with mean zero and variance \( \sigma^2 \), and the noises are statistically independent; therefore, \( E[N_x(t)N_x(t)^H] = \sigma^2 I \), \( E[N_y(t)N_y(t)^H] = \sigma^2 I \). The matrix \( P = E[S(t)S(t)^H] \) is positive definite.

From this signal model, we can see that we did not consider the multipath effect of users. Therefore, the mobile communication environment used in this model is an open suburban area or a rural area [5].

The auto-correlation matrix of signal \( X(t) \) is expressed as
\[ R_{xx}(t) = E[X(t)X(t)^H], \]
the cross-correlation matrix of signal \( X(t) \) and \( Y(t) \) is expressed as \( R_{yx}(t) = E[Y(t)X(t)^H], \) using the formulas (3) and (4), we can get
\[ R_{xx}(t) = APA^H + \sigma^2 I, \quad R_{yx}(t) = A\Phi PA^H + \Omega. \]

Where, \( P = E[S(t)S(t)^H] \). If \( L > M \), then all the elements of \( \Omega \) are zero. Otherwise, except the element \((1, L), (2, L+1), (M-L-1, M)\) are \( \sigma^2 \), the elements remaining are zero. We will change the \( \Omega \) into zero matrix according to the estimated noise variance (refer to Algorithm Step 3), so in the derivation below the \( \Omega \) will be omitted.

After the assumption and treatment, we can use the method and principle in document[2] to get the following formula:
\[ R_{yx} \hat{R}_{xx0} ^* A = A\Phi \quad (5) \]

Where, \( R_{xx0} = APA^H, \hat{R}_{xx0} ^* \) express the pseudo inverse matrix[6], which can be expressed as
$$R_{xx0}^* = \sum_{i=1}^{K} \frac{1}{\mu_i - \sigma^2} V_i V_i^H,$$

where, $\mu_i$ and $V_i$ are the eigenvalue and the eigenvector of the $R_{xx}$ respectively. Since matrix $\Phi$ is a diagonal matrix, equation (5) suggests that column vectors of the array manifold $A$ is the eigenvectors of matrix $R_{xx} R_{xx0}^*$, and the diagonal elements corresponding to the diagonal matrix $\Phi$ is eigenvalue of the matrix $R_{xx} R_{xx0}^*$. Let $R = R_{xx} R_{xx0}^*$, and call that the DOA matrix[2] (as can be seen from the equation above, $R$ has only $K$ non-zero eigenvalues[1]).

**DOA ALGORITHM BASED ON SIGNAL SPACE FEATURES**

According to the eigenvalue decomposition of DOA matrix, we can get the DOA estimation algorithm like following: (1) According to the received signal, estimating the auto-correlation matrix $R_{xx}$ of $X(t)$ and cross-correlation matrix $R_{yx}$ of $X(t), Y(t)$. In this paper, we use the progressive unbiased estimator[7]:

$$R_{xx} = \frac{1}{N} \sum_{i=1}^{K} X(t) X(t)^H, R_{yx} = \frac{1}{N} \sum_{i=1}^{K} Y(t) X(t)^H.$$ (2) Calculating the eigenvalue ($\mu_1, \mu_2, \ldots, \mu_N$) and eigenvector ($(V_1, V_2, \ldots, V_N)$) of $R_{xx}$; (3) From formula (18), we can see that the user’s number $K$ is the larger number in $\mu_1, \mu_2, \ldots, \mu_N$ (the smaller eigenvalue corresponds to the variance of the noise, so the variance of noise can be taken as the average of the smaller eigenvalues so that the $\Omega$ in the formula (8) can be changed into zero matrix); (4) Constructing DOA matrix $R$ according to the formula (19); (5) Calculating the eigenvalues and eigenvectors of $R$, from which selecting $K$ larger eigenvalues $\epsilon_i$ and their corresponding eigenvectors $Z_i$; (6) Constructing search function $f(\theta) = \frac{1}{N} \sum_{i=1}^{K} \frac{1}{\|a(\theta) - \frac{Z_i}{Z_i(1)}\|^2}, \quad i = 1, 2, \ldots, L$, where, $Z_i(1)$ is the first element of the eigenvector $Z_i$. The angle $\theta$ corresponding to the peak value of $f(\theta)$ is the arrival angle of the user $i$.

**NUMERICAL SIMULATION**

For this algorithm, we conducted a numerical simulation to verify its validity. Test one: this test is used to verify the algorithm and estimate the performance of multi-user signal DOA. Array element is 9, user’s number is 4, received signal is BPSK signal, carrier frequency is $f_c = 900MHz$, signal’s baud rate is $2f_c$, sampling points is 2000, signal-noise ratio is 20dB. The angle of incidence for the 4
users is (-60, -50, 20, 40). The results are shown as the Fig.1; the vertical axis of the figure is in units of dB, the peak corresponding to the abscissa is the user’s DOA. A total of 10 trials were conducted. Test two: this test is used to verify the resolution ratio. User’s number is 2, incidence angle is (-2, 2). The remaining parameters are consistent with the test one, and the results is shown in Fig.2.

CONCLUSIONS

In this paper, a straight linear array is divided into two identical sub-arrays, and then estimating the DOA according to the array signal spatial structure. In this method, the eigenvalue decomposition feature of signal’s auto-correlation and cross-correlation matrix is fully used to construct DOA matrix of array signal, the eigenvector corresponding to the non-zero eigenvalue of this matrix does is array manifold, and the user’s arrival angle can be obtained through searching the extreme value of function. The results of computer simulation suggests this method is feasible and the resolution ratio is also high.

REFERENCES