Modeling on Distance Error of Bistatic Sonar Detection

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ABSTRACT

Based on the principle of bistatic sonar orientation, we build the model of estimation on distance error considering the measured error of time, direction and acoustic velocity, and detailedly analyse the mean square error influenced by centrifugal rate of ellipse and orientation. The outcome of simulation indicate that the orientation and position of the targets apart from the sender and receiver impact the estimated precision of distance. On the condition of the distance of sender and receiver fixed, the position more apart from the sender and receiver, the distance estimation lower effect influenced by the orientation of targets, and the value of mean square error of distance keeps in a lower scope. According to the model of estimation presented by this paper, deploy the bistatic sonar reasonably can realized high precision and detected targets in a long distance.

INTRODUCTION

Active sonar can get the exact location of the target, but the detection distance is limited for the two-way propagation loss. So, it is necessary for single base sonar to improve the transmitting power in order to obtain a longer detection range. This requires increasing the size of the sonar array, and also will reduce the concealment of the sonar. Passive sonar is well concealed, but the detection capability is limited by the target radiated noise and it is difficult to locate the target. The bistatic sonar combines the advantages of active sonar and passive sonar, for the receiving end is
far away the transmitting platform, this can avoid reverberation and other strong interference, so, the bistatic sonar has the advantages of good concealment and high location accuracy.

Bistatic sonar has been widely used in military domain, such as the American Jolly system which has been used from the 1960s [1]. Passive sonar receives the echo from a submarine generated by an air-dropped bomb and then locate the submarine in a long distance. With the development of equipment technology, the detection capability of such sonar system has been greatly improved. The researches on bistatic sonar have never stopped, Yisheng Yan simply estimated the sonar detection distance based on sonar equation in 1996 [2]. Li Yang deduced the maximum detectable range of bistatic sonar according to the energy relationship and researched on the sonar configuration [3]. Xiaofeng Zhang researched the effect of time measurement error on the accuracy of sonar location based on TOL algorithm [4]. However, systematical research on the distance measurement error of bistatic sonar is very little. In order to further study the locating performance of bistatic sonar, in this paper, the bistatic sonar distance estimation error model is built based on the work principle, and the main influencing factors of sonar detection accuracy is analyzed. This provides a theoretical basis for further study of bistatic sonar detection performance.

Location Principle of Bistatic Sonar

The configuration of the bistatic sonar is shown in the figure 1. One ship transmits active sonar signals, the signals are reflected by the target and then received by the array fixed on the other boat. Set the sound signal transmission distance is $X$, which includes the distance $\sigma$ from sound source to the target and the distance $R$ from the target to the receive boat. In fact, the first arrival to the receiving array is direct sound waves, the sound waves are transmitted from the sound source and propagated directly from the distance $S$ to the receiving vessel, distance $S$ is called the reference length. The parameter $\gamma$ is the angle between the target and the receiving ship against the baseline. In this paper, we call it azimuth, which is usually determined by the target orientation and the transmitting and receiving position. From the geometric relationship shown in Figure 1 we can derive expression (1).

\[
(X - R)^2 = R^2 + S^2 - 2RS \cos \gamma
\]  

(1)

When the target is within the detecting range of the bistatic sonar, if the distance $S$ and the sound wave propagation distance $X$, and then get the azimuth $\gamma$ will be able to determine the target distance list as expression (2).
\[ R = \frac{X^2 - S^2}{2(X - S \cos \gamma)} \]  

(2)

\[ X = \sigma + R \]

Figure 1. Deployment of bistatic sonar.

In order to make full use of the advantage of low noise level and good concealment of bistatic sonar, the receiving vessel is generally far away from the transmitting vessel and the target, the distance \( S \) and \( X \) need real-time measurements. Set the time of the sound wave propagates from the source to the receiving array along the reference line \( S \) is \( t_s \), the time of the sound wave propagates along the line \( X \) is \( t_x \), then \( X = V t_x \), \( S = V t_s \), the parameter \( V \) is the average propagation speed of the sound. Usually, it is difficult to measure the time \( t_x \) directly, but the time delay \( t_x \) and \( t_s \) is easy to get by expression (3).

\[ t_x = t + t_s \]  

(3)

So, when the time delay \( t \) of echo and direct wave are measured, the propagation time \( t_x \) can be calculated by (3). In the case of the azimuth \( \gamma \) and the speed \( V \) are known, the distance from the target and the receiving vessel can be given by expression (4).

\[ R = \frac{V (t_x^2 - t_s^2)}{2(t_x - t_s \cos \gamma)} \]  

(4)

**Model of Range Error**

Due to the complexity of marine underwater acoustic environment, the sound speed is uncertainly in the ocean, it is influenced by time and space. When the sounds propagate along different path, the value of sound speed is also different.
When we calculate the distance by expression (4), there is error caused by the variety of sound speed. Set the average speed of the sound propagating along the indirect path $X$ is $U$, then $X = Ut_x$, we get the more reasonable expression list as expression (5).

$$R = \frac{(Ut_x)^2 - (Vt_s)^2}{2(Ut_x - Vt_s \cos \gamma)}$$  \hspace{1cm} (5)

Set the difference of the sound speed propagating along two paths as $\tau_x = \frac{U}{V}t_x$, then expression (5) can be expressed as follow.

$$R = \frac{V(\tau_x^2 - \tau_s^2)}{2(\tau_x - \tau_s \cos \gamma)}$$  \hspace{1cm} (6)

In fact, if the three parameters except $\gamma$ in expression (6) are known, then we can draw the conclusion that the target is located on an ellipse, the focus of the ellipse are the positions of the transmitting vessel and receive vessel, the centrifugal rate of the ellipse $e$ is given by expression (7).

$$e = \frac{S}{X} = \frac{Vt_s}{Ut_x} = \frac{t_s}{\tau_x}$$  \hspace{1cm} (7)

This ellipse is called positioning ellipse. In uniform seawater, $U = V$, the centrifugal rate of the ellipse depends on the ratio of the time of sound propagating along path $S$ and path $X$, when the distance between the transmitting vessel and the receiving vessel is fixed, the centrifugal rate of the ellipse depends on the sum of the distances from the target to the transmitting vessel and to the receiving vessel. If the position ellipse is fixed, we can determine the target position by the ellipse and the azimuth $\gamma$. Figure 2 and Figure 3 shows the influence of ellipse centrifugal rate and the azimuth to the target distance. By expression (6) and expression (7), we can calculate it by expression (8).

$$R = \frac{Vt_s(1 - e^2)}{2e(1 - e \cos \gamma)} = \frac{S}{2e(1 - e \cos \gamma)}$$  \hspace{1cm} (8)
Expression (6) shows that the main factors which affect the accuracy of bistatic sonar distance measurement are $V$, $\tau_x$, $t_s$, $\gamma$, the relative error of target distance depends on $\Delta V$, $\Delta \tau_x$, $\Delta t_s$, $\Delta \gamma$.

$$\frac{\Delta R}{R} = \frac{1}{R} \frac{\partial R}{\partial V} \Delta V + \frac{1}{R} \frac{\partial R}{\partial \tau_x} \Delta \tau_x + \frac{1}{R} \frac{\partial R}{\partial t_s} \Delta t_s + \frac{1}{R} \frac{\partial R}{\partial \gamma} \Delta \gamma$$

(9)

The parameters of expression (7) can be calculated by expression (6).

$$\begin{aligned}
&\frac{1}{R} \frac{\partial R}{\partial V} = \frac{1}{V} \\
&\frac{1}{R} \frac{\partial R}{\partial \tau_x} = \frac{2 \tau_x}{\tau_x^2 - t_s^2} - \frac{1}{\tau_x - t_s \cos \gamma} \\
&\frac{1}{R} \frac{\partial R}{\partial t_s} = \frac{-2 \tau_x}{\tau_x^2 - t_s^2} + \frac{\cos \gamma}{\tau_x - t_s \cos \gamma} \\
&\frac{1}{R} \frac{\partial R}{\partial \gamma} = \frac{t_s \sin \gamma}{\tau_x - t_s \cos \gamma}
\end{aligned}$$

(10)

Next, these coefficients are substituted into expression (7) to derive the expression that calculates the relative error of target distance.

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \left( \frac{2 \tau_x}{\tau_x^2 - t_s^2} - \frac{1}{\tau_x - t_s \cos \gamma} \right) \Delta \tau_x - \left( \frac{2 t_s}{\tau_x^2 - t_s^2} - \frac{\cos \gamma}{\tau_x - t_s \cos \gamma} \right) \Delta t_s - \frac{t_s \sin \gamma}{\tau_x - t_s \cos \gamma}$$

(11)

Define the parameters in expression (11) as:

$$\tau_x = \frac{U}{V} t_x, \Delta \tau_x = \frac{U}{V} \Delta t_x, \frac{\Delta t_x}{\tau_x} = \frac{\Delta t_s}{t_x}, \frac{V}{U} \frac{\Delta t_s}{t_x}$$

Substitute these expressions into expression (9), then we can get another expression.
\[ \frac{\Delta R}{R} = \frac{\Delta V}{V} + \left( \frac{2}{1-e^2} - \frac{1}{1-e \cos \gamma} \right) \frac{\Delta t_x}{t_x} + \left( \frac{-2e^2}{1-e^2} - \frac{e \cos \gamma}{1-e \cos \gamma} \right) \frac{\Delta t_y}{t_y} - \frac{e \sin \gamma}{1-e \cos \gamma} \Delta \gamma \]  \quad (12)

In engineering measurement, we often use the root mean square error to measure the deviation between the measured value and the true value. If the measurement values and errors of \( V, t_x, t_y, \gamma \) are known, we can then calculate the relative error of the target distance by expression (12). On the other hand, if the measurement errors of the parameters in expression (12) are statistical independent, in case of other measurement error is stable, we can discuss the influence of one measurement error on the distance error. When each parameter error is statistical independent, the root mean square error can be reduced to the sum the squares of each independent parameter errors and then extract a root. Define the error coefficients as expression (13).

\[
\begin{align*}
A_x &= \frac{2}{1-e^2} - \frac{1}{1-e \cos \gamma} \\
A_y &= \frac{-2e^2}{1-e^2} + \frac{e \cos \gamma}{1-e \cos \gamma} \\
A_\gamma &= -\frac{e \sin \gamma}{1-e \cos \gamma}
\end{align*}
\quad (13)
\]

The root mean square error model of the target distance can be expressed as expression (14).

\[
\frac{1}{R} (\Delta R)_{rms} = \sqrt{\frac{\Delta V^2}{V^2} + A_x^2 \left( \frac{\Delta t_x}{t_x} \right)^2 + A_y^2 \left( \frac{\Delta t_y}{t_y} \right)^2 + A_\gamma^2 (\Delta \gamma)^2} \quad (14)
\]

It can be seen from (14) that the factors which affect the root mean square error of distance include the relative errors of the measured values and error coefficients. The relative error of the measured values includes fixed and random error, the fixed error depends on the accuracy of the instrument manufacturing, for a given instrumentation, this part of the error can not be eliminated. Random errors are generated by human operations, this part of the error can be reduced by increasing the operating level. Generally, the relative measurement error changes randomly in a small scale [5], there is little effect on the root mean square error of distance. However, the effect of the error coefficient on the root mean square error of distance is much greater, by the expression (13) we can see that the error coefficients depend on the values of centrifugal rate of the positioning ellipse and the azimuth. Figure 4 shows the relationship of error coefficients absolute value and azimuth in different centrifugal rate of ellipse.
Figure 4 shows that the error coefficients change differently with the change of azimuth and ellipse centrifugal rate. When the centrifugal rate is relatively small, the error coefficients change slowly with the azimuth, they increase together with the ellipse centrifugal rate. The value of $X_A$ is an incremental function of parameter $e$ and parameter $\gamma$. The value of $|A_s|$ and $|A_e|$ have maximum and minimum values at certain ellipse centrifugal rates. Further, if we takes more discrete values of the centrifugal rate in the range of (0,1), then we can draw the following conclusions.

(1) The more smaller the centrifugal rate is, which means the ellipse is more flat, the values of $|A_x|$, $|A_s|$, $|A_e|$ change more slowly with the change of $\gamma$. And when the values of $|A_s|$ and $|A_e|$ are closer, the values of $|A_x|$ is bigger.

(2) The more smaller the centrifugal rate is, which means the ellipse is more round, the values of $|A_x|$, $|A_s|$, $|A_e|$ change more quickly with the change of $\gamma$. As the value of $\gamma$ increasing, the values of $|A_x|$ and $|A_s|$ increase firstly and then
gently and become consistent at last, but the value of $|A_c|$ increase firstly and then decrease to zero at last.

(3) Regardless of the value of the centrifugal rate, the value of $|A_x|$ is bigger than the values of $|A_3|$ and $|A_c|$ in most part of the range of $\gamma$. In summary, for bistatic sonar, the azimuth of target and the distance from target to the transmitting and receiving vessel influence the range measurement accuracy markedly. When the distance from the transmitting vessel to the receiving vessel is fixed, the target gradually away from the transmitting and receiving vessel, the effect of azimuth on distance measurement error is reduced, the root mean square error is also maintained at a low level. This fully shows that the bistatic sonar long-range detection has a high locating accuracy.

CONCLUSIONS

Based on the principle of bistatic sonar location, we build the model of distance estimation error by considering the measured error of time, direction and acoustic velocity, and analyze the mean square error influenced by centrifugal rate of ellipse and orientation in detail. The outcome of simulation indicates that the orientation and position of the target against the sender and receiver influence the distance precision markedly. So, it is possible for bistatic sonar by deploying a reasonable sender and receiver position to detect a long distance target with higher locating accuracy.

REFERENCES