The Direct Quaternion Method in Initial Alignment of SINS

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Abstract. In this paper, the initial quasi-quaternion method of strapdown inertial navigation system (SINS) is proposed, in order to solve the singularity problem of the initial alignment algorithm. And mathematical simulations is conducted and analysed. Simulation results show that the method is feasible.

Introduction

The purpose of the initial alignment of the strap down inertial navigation system (SINS) is to establish the initial value of the strap down matrix. The initial alignment error is one of the main error sources of the SINS, which directly affects the precision of the inertial navigation system (INS). At present, there are three main methods for calculating the attitude angle of the SINS, that is the direction cosine method, the Euler angle method and the quaternion method. The directional cosine method, which solves the direction of the cosine differential equation, needs to solve the nine significant computational differential equations. It is not practical in engineering. The Euler angle method can be directly to the three attitude angle. But it depends on the rotation order, and it cannot work in all pose. Having advantages of relatively small amount of calculation, the quaternion method has been widely used in engineering. At present, the method commonly used in engineering is based on the initial alignment method of the quintuple of Euler angles. The method needs to find the Euler angle first, and then calculation of the initial quaternion. There is a singularity in solving the Euler angles. In this paper, an initial alignment method is investigated to solve this problem. This method avoids solving the Euler angles and can obtain the quaternion directly through the quaternion attitude transformation matrix.

Initial Alignment method for SINS

For a SINS, the key to the initial alignment lies in the precise pursuit of the high-precision strap down attitude matrix. The attitude matrix in the strap down inertial navigation system is equivalent to making a coordinate transformation on the platform system. This mapping plays a platform role. Therefore, the attitude matrix can also be regarded as a "mathematical platform". Attitude matrix is the function of the attitude angles named $\psi$, $\vartheta$, $\gamma$, which are formed in accordance with a certain Euler angle sequence on the rotation of the coordinate system.

The transformation from the navigation coordinate system $O \textsuperscript{n}X \textsuperscript{n}Y \textsuperscript{n}Z \textsuperscript{n}$ to the projectile coordinate system $O \textsuperscript{p}X \textsuperscript{p}Y \textsuperscript{p}Z \textsuperscript{p}$ is achieved by successive rotation (without considering the movement of the origin of the coordinate): the First, rotate around the axis $O \textsuperscript{n}Y$ by the angle $\psi$. Then rotate around the new axis $O \textsuperscript{p}Z$ by the angle $\vartheta$, and then around new axis $O \textsuperscript{p}X$ (ie, the axis $O \textsuperscript{p}X$) by the angle $\gamma$. $\psi$, $\vartheta$, $\gamma$ are named yaw, pitch and roll angles respectively. In this way, the transformation matrix from the navigation coordinate system to the missile coordinate system is:
With the elements of the above matrix we can determine the main value of the attitude angle $\vartheta$, $\psi$, $\gamma$. The commonly used acquisition methods of the attitude matrix are numerous including used direction cosine method, Euler angle method, quaternion method and rotation vector method, etc. In order to reduce the computational complexity and avoid the singularity. In this paper, a direct quaternion method is proposed to calculate the attitude matrix.

**Quaternion Initial Alignment Method**

**Coordinate System Definition**

The coordinate system involved in this paper is as follows:

a) the navigation coordinate system $(O_nX_nY_nZ_n)$

Origin $O_n$: located in the launch frame positioning point;
Axis $O_nX_n$: with the point through the tangent line of heavy, pointing to the north is positive;
Axis $O_nZ_n$: with the point through the weft tangent, point to the east is positive;
Axis $O_nY_n$: along the origin of the vertical line, pointing to the positive;
After the launch with the earth, with the earth together.

b) the missile coordinate system $(O_bX_bY_bZ_b)$

Origin $O_b$: located in the missile center of mass;
Axis $O_bX_b$: along the longitudinal axis of the missile, pointing to the missile head is positive;
Axis $O_bY_b$: in the longitudinal plane of the missile, perpendicular to the axis $O_bX_b$, pointing to the positive;
Axis $O_bZ_b$: according to the right hand rule to determine.

**Relationship between Quaternion and Attitude Angle**

The quaternion, which represents the attitude matrix from the navigation coordinate system to the missile coordinate system is:

$$
\mathbf{C}_n^b = \begin{bmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_0q_3 - q_1q_2) \\
2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_0q_1 - q_2q_3) \\
2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2
\end{bmatrix}
$$

The attitude angle is defined as the order of the first yaw and then the last roll. The attitude matrix from the navigation coordinate system to the missile coordinate system is:

$$
\mathbf{C}_n^b = \begin{bmatrix}
\cos \vartheta \cos \psi & \sin \vartheta & -\cos \vartheta \sin \psi \\
-\sin \vartheta \cos \gamma + \sin \gamma \sin \vartheta & \cos \vartheta \cos \gamma & \sin \gamma \cos \vartheta \sin \psi + \sin \gamma \cos \psi \\
\sin \vartheta \cos \gamma \sin \psi + \cos \gamma \sin \vartheta & \sin \gamma \cos \psi & -\sin \vartheta \sin \psi \cos \gamma + \sin \gamma \cos \psi
\end{bmatrix}
$$
Initial Alignment Method Using the Quaternion

In the initial attitude, the initial alignment equation is:

\[
\begin{bmatrix}
\frac{f_x}{g} \\
\frac{f_y}{g} \\
\frac{f_z}{g}
\end{bmatrix} = C^b_n \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]  

(4)

Then

According to the relationship between the quaternion and the attitude angle, the equation is:

The equation of direct quaternion method is:

\[
\begin{cases}
q_0 = \frac{1}{2} \sqrt{f_x^2 + f_y^2 + f_z^2} \left(1 - \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \cos \frac{\psi}{2} - \frac{1}{2} \sqrt{f_x^2 + f_y^2 - f_z^2} \left(1 - \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \sin \frac{\psi}{2} \\
q_1 = \frac{1}{2} \sqrt{f_x^2 + f_y^2 - f_z^2} \left(1 + \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \cos \frac{\psi}{2} + \frac{1}{2} \sqrt{f_x^2 + f_y^2 + f_z^2} \left(1 - \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \sin \frac{\psi}{2} \\
q_2 = \frac{1}{2} \sqrt{f_x^2 + f_y^2 - f_z^2} \left(1 - \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \cos \frac{\psi}{2} + \frac{1}{2} \sqrt{f_x^2 + f_y^2 + f_z^2} \left(1 + \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \sin \frac{\psi}{2} \\
q_3 = \frac{1}{2} \sqrt{f_x^2 + f_y^2 + f_z^2} \left(1 - \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \cos \frac{\psi}{2} - \frac{1}{2} \sqrt{f_x^2 + f_y^2 - f_z^2} \left(1 + \sqrt{1 - \left(\frac{f_z}{g}\right)^2}\right) \sin \frac{\psi}{2}
\end{cases}
\]  

(7)

where \(\psi\) is obtained by binding.

Mathematical Simulation

The specific conditions and simulation results of the initial alignment simulation condition are shown in Table 1. Simulation of the relevant curve shown in Figure 1 to Figure 4.

Table 1. Specific conditions and simulation results.

<table>
<thead>
<tr>
<th>Simulation conditions</th>
<th>Attitude angle</th>
<th>character</th>
<th>paragraph</th>
</tr>
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<tbody>
<tr>
<td>set0</td>
<td>-0.6</td>
<td>-0.646</td>
<td></td>
</tr>
<tr>
<td>psi0</td>
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<td>-90</td>
<td></td>
</tr>
<tr>
<td>gam0</td>
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<td>-0.299</td>
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</table>
Figure 1. Initial alignment pitch curve.  
Figure 2. Initial alignment yaw curve.  
Figure 3. Initial alignment of the roll angle curve.
Conclusion
Aiming at solving problems of pose angle singularity in the initial alignment of SINS, a quaternion alignment method is proposed. The calculation amount of quaternion method is small and there is no singularity. The initial alignment method of quaternion is obtained directly by quaternion attitude transformation matrix, which can improve both the efficiency of initial alignment and apply larger attitude angle range.

References