A Study of Unscented Kalman Filter Performance in the Ultralight Coupled Integration System

YUNPENG FAN and SUOCHANG YANG

ABSTRACT

This paper aims to the enhancement of the performance in the navigation filter of the GNSS/INS ultralight coupled (UTC) integration system. The basic structure of the UTC and the state-space equations are provided in this paper. Owing to its strong nonlinearity, a kind of nonlinear Kalman filter based on the unscented transformation is introduced to the system, which can lead to higher accuracy and stronger robustness than traditional EKF. Furthermore, when applying into a realistic nonlinear system without detailed information about the model and noise, an advanced UKF is proposed in this paper to deal with the potential increasing state-error and divergence of the system. Finally, Simulations are made to prove the availability of these theories above.

KEYWORDS

GNSS/INS integration, Ultralight coupled integration system, Unscented Kalman filter

INTRODUCTION

Global Navigation Satellite System (GNSS) can provide highly accurate position information, and the errors of data won’t accumulate over time. However, it can be easily interrupted by electromagnetic waves and other noise, which makes it impossible for the system to provide precise position data persistently. Inertial Navigation System (INS) has good performances in many aspects. It can work smoothly with seldom hardware failure, and the process could remain stable even though in a complicated environment. The INS integrates three acceleration components and three angular velocity components to obtain the position and velocity information, meanwhile the errors of data will be accumulated during the integration operation. Owing to the inherent complementary of those two systems, researchers have sought for the combination of them naturally to reduce the weaknesses and strengthen the advantages of both. The GNSS/INS integrated navigation system performs well in providing continuous, high bandwidth, complete and long-time precise position information, and it has become a hotspot in the area of navigation in the recent years [1].

Yunpeng Fan, Department of Missile Engineering, Army Engineering University, Shijiazhuang 050003, China; fyp_940630@foxmail.com.
Suochang Yang, Department of Missile Engineering, Army Engineering University, Shijiazhuang 050003, China; yscyse2003@sina.com.
There are three typical kinds of integrated modes in GNSS/INS integrated navigation system which are named loosely coupled, tightly coupled and ultra-tightly coupled for each. The ultra-tightly coupled (UTC) structure combines the correlate components I (in-phase) and Q (quadrature) signals and the INS navigation filter into one single integrated filter, described as tracking area integration. Although it is much more difficult for the UTC structure to be implemented, the system has greater dynamic performance and better anti-jamming ability in complicated conditions [2].

The design of integrated filter has a great influence on the performance of the UTC system. Due to the strong nonlinearity of the UTC system, the common Kalman Filter doesn’t make sense to accomplish the mission. Researchers had proposed some other ways to solve the problem, such as using some special Kalman Filters like the Extended Kalman Filter (EKF) or the Unscented Kalman Filter (UKF). The EKF method has been proved to be the filter that minimizes the variance of the estimation mean square error (MSE). However, it heavily depends on a predefined dynamics model. A complete priori knowledge of the dynamic process and measurement models is required to achieve desired results. In most situations, the information above cannot be easily acquired, which might lead to the unavailability of the EKF method or a divergence problem. The UKF method is proposed by Julie and Ullman [3-5] to overcome the deficiencies of the EKF method. The UKF method has the similar frame as the EKF. It makes a Gaussian approximation with a series of points (named as sigma points) through the Unscented Transformation (UT) instead of linearization directly, which avoids the difficulties in the linearization of strong nonlinear functions. Transformed by the nonlinear system function, the posterior mean and variance captured by the sigma points can be as precise as third-order accuracy or higher of the nonlinear system’s Taylor expansion[], which is more precise than the EKF. Due to the advantages above, the UKF method has been widely acknowledged as a perfect alternative to the EKF method.

This paper consists of the following parts. In Sect. 2, the basic structure of the UTC system is introduced. The basic UKF method is reviewed in Sect. 3. In Sect. 4, the usage of UKF method in UTC system will be applied. Conclusions and discussions are given in Sect. 6.

THE ULTRATIGHTLY COUPLED GNSS/INS INTEGRATED NAVIGATION SYSTEM MODULE

The typical structure of the UTC system [6] is shown in Fig. 1. The red lines in the figure show some important signal flow in this structure. The RF front-end receives satellite signals and outputs the IF sampling signal which can be written as

\[ s_{IF}(t) = \sqrt{2P_s}C(t-\tau) \cdot D(t-\tau) \cos(\omega_{IF}t + \phi(t)) + n(t) \]  \hspace{1cm} (1)

Where \( P_s \) is the power of the signal; \( C(t) \) is the C/A code; \( \tau \) is the time delay in the transmission process; \( D(t) \) is the navigation message bits, whose bit rate is 50 bps; \( \omega_{IF} \) is the frequency of the intermediate-frequency carrier; \( \phi(t) \) is the initial
carrier phase; \(n(t)\) is the white noise whose power spectrum density (PSD) is considered as constant, represented as \(\frac{N_o}{2}\).

The correlator component generates local I/Q signals to multiply by the IF sampling signals and integrates them after that. Suppose the integration time is \(T_i\), then we have

\[
I = \int_{kT_i}^{(k+1)T_i} \sqrt{P_s} C(t - \tau) C(t - \hat{\tau}) D(t - \tau) \cos(\omega_e t + \phi_e) dt + \tilde{N}_I \tag{2}
\]

\[
Q = \int_{kT_i}^{(k+1)T_i} \sqrt{P_s} C(t - \tau) C(t - \hat{\tau}) D(t - \tau) \sin(\omega_e t + \phi_e) dt + \tilde{N}_Q \tag{3}
\]

where \(\omega_e\) and \(\phi_e\) is the frequency and phase error between local I/Q signals with the IF sampling signals respectively; \(C(t - \hat{\tau})\) is the local pseudo code; \(T_i\) is often set as an integral multiple of the C/A code period and less than 20ms. During that time \(D(t)\) is constant, hence it can be moved out of the integral sign. It can be considered that \(\tau = \hat{\tau}\) when the signal tracking loop operates in a stable way. Then we have

\[
I = \frac{D\sqrt{P_s}T_i}{\omega_e} \left[ \sin(\omega_e (k+1)T_i) + \phi_e \right] - \sin(\omega_e kT_i + \phi_e) + \tilde{N}_I \tag{4}
\]

\[
Q = -\frac{D\sqrt{P_s}T_i}{\omega_e} \left[ \cos(\omega_e (k+1)T_i) + \phi_e \right] - \cos(\omega_e kT_i + \phi_e) + \tilde{N}_Q \tag{5}
\]
From the Fig. 1 we can learn that the measurements of the GNSS/INS integrated filter is
\[ z_i = \left[ dI + \vec{N}_i, dQ + \vec{N}_Q \right], \quad i = 1, 2, ..., m, \]

Where \( m \) represents the amount of the tracked satellite channels. To establish the measurement equation of the UTC model, we are supposed to find the relations between the I/Q integral and the inertial sensor measurements. According to the Doppler Principle, we have
\[ \omega_e = \frac{\omega_{\text{IF}}}{c} V_e \]  
\[ \phi_e = \frac{\omega_{\text{IF}}}{c} (V_e t - R_e) \]

Where \( V_e \) and \( R_e \) represents the error of velocity and position respectively; \( c \) is the velocity of light. Furthermore, \( V_e \) and \( R_e \) can be represented as
\[ V_e = \sqrt{\dot{x}_e + \dot{y}_e + \dot{z}_e} \]  
\[ R_e = \sqrt{x_e + y_e + z_e} \]

Through taking expectation of the I/Q integral we can eliminate the effects of the noise. Based on the analysis above, the relations between the I/Q and the R/V are given as
\[ dE(I) = h_{ix} dx + h_{iy} dy + h_{iz} dz + h_{ix} \dot{x} + h_{iy} \dot{y} + h_{iz} \dot{z} \quad (10) \]
\[ dQ(I) = h_{qx} dx + h_{qy} dy + h_{qz} dz + h_{qx} \dot{x} + h_{qy} \dot{y} + h_{qz} \dot{z} \quad (11) \]

The coefficient \( h \) can be expressed as
\[ h_{ix} = \frac{1}{2} \left( \frac{\partial E(I)}{\partial \phi_e} \frac{\partial \phi_e}{\partial x} + \frac{\partial E(I)}{\partial \omega_e} \frac{\partial \omega_e}{\partial x} \right) dx \quad (12) \]
\[ h_{qx} = \frac{1}{2} \left( \frac{\partial Q(I)}{\partial \phi_e} \frac{\partial \phi_e}{\partial x} + \frac{\partial Q(I)}{\partial \omega_e} \frac{\partial \omega_e}{\partial x} \right) dx \quad (13) \]

The rest of them for y and z component has the same form with the formula (12) and (13). The detailed expressions are referred to in [7]. With the aid of these coefficients we can establish the measurement matrix as
We choose a 17-dimensional vector to be the state vector of the GNSS/INS integrated navigation filter. It consists of three inertial components each in position, velocity, attitude, accelerometer bias, gyroscope bias, and two other states for the receiver clock bias and drift

\[
x_k = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} & \phi_x & \phi_y & \phi_z & a_x & a_y & a_z & g_x & g_y & g_z & b & d \end{bmatrix}^T_{17}
\]

Now we have accomplished the establishment of the state-space module for the GNSS/INS integrated navigation filter, which is the most critical component in the UTC system, building the foundation of the discussion in the following sections.

THE BASIC PRINCIPLES OF THE UNSCENTED FILTER

A nonlinear filter problem can be described as state equations below

\[
\begin{align*}
x_k &= f(x_{k-1}) + w_k, \\
z_k &= h(x_k) + v_k
\end{align*}
\]

Given that

\[
\begin{align*}
E[w_i w_j^T] &= \begin{cases} Q_k, & i = k \\
0, & i \neq k \end{cases} \\
E[v_i v_j^T] &= \begin{cases} R_k, & i = k \\
0, & i \neq k \end{cases} \\
E[w_i v_j^T] &= 0, \text{ For all } i \text{ and } k
\end{align*}
\]

The Unscented Transformation

Julie and Ullman [3-5] thought it is easier to approximate the probability density distribution of the nonlinear function than approximating the function itself. That is the reason they proposed the UT method. The UT can be described as follows [8].

For n-dimensional random variable \( x \) with the mean value \( \bar{x} \) and the covariance \( P_x \)
, we can generate \( 2n + 1 \) sigma points \( \tilde{x} \) as
Weights for each sigma points are given as

\[
\begin{cases}
  w^m_i = w^c_i = \frac{\kappa}{n + \kappa} & i = 0 \\
  w^m_i = \frac{1}{2(n + \kappa)} & i \neq 0
\end{cases}
\]  

(20)

Where \( \kappa \) is a free parameter to control the distance between every point to the mean value, and it is often set as \( \kappa = 3 - n \). \( w^m_i \) is the weight for the mean value corresponding to the itch point; \( w^c_i \) is the weight for the covariance corresponding to the itch point. The weights satisfy the equation \( \sum w^m_i = \sum w^c_i = 1 \).

Suppose the sigma points propagate through a nonlinear transformation function \( z = f(x) \), then we have

\[
z_i = f(\tilde{x}_i)
\]  

(21)

The mean and covariance can be calculated by

\[
\bar{z} = \sum_{i=0}^{2n} w^m_i z_i
\]  

(22)

\[
P_z = \sum_{i=0}^{2n} w^c_i (z_i - \bar{z})(z_i - \bar{z})^T
\]  

(23)

The Unscented Kalman Filter Process

The implementation of the UKF process can be described as several steps as follows:

1. Initialize the state estimation \( \hat{x}_0 \) and the covariance matrix \( P_0 \);
2. For \( k \geq 1 \)
   (1) Decompose the covariance matrix \( P_{k-1|k-1} \) in Chelsey decomposition method:

\[
P_{k-1|k-1} = S_{k-1|k-1} S_{k-1|k-1}^T
\]  

(24)
(2) Calculate the sigma points $X_{i,k|k-1}$ using equation (19)

$$X_{i,k|k-1} = S_{k|k-1} \eta_i + \hat{x}_{k-1|k-1}, i = 1, \ldots, 2n$$  \hspace{1cm} (25)

Where $\eta_i = \sqrt{n+k} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $[1]$ is the $i$th column of $[-I_n \ I_n]$;

(3) Propagate the sigma points through state function

$$X_{i,k|k-1}^* = f\left(X_{i,k|k-1}\right)$$  \hspace{1cm} (26)

(4) Predict the $k$th state

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m X_{i,k|k-1}^*$$  \hspace{1cm} (27)

(5) Predict the $k$th state error covariance matrix

$$P_{k|k-1} = \hat{x}_{k|k-1} - \sum_{i=0}^{2n} w_i^m \left(X_{i,k|k-1}^* - \hat{x}_{k|k-1}\right)\left(X_{i,k|k-1}^* - \hat{x}_{k|k-1}\right)^T + Q_{k-1}$$  \hspace{1cm} (28)

(6) Decompose $P_{k|k-1}$ in Chelsey decomposition method:

$$P_{k|k-1} = S_{k|k-1} S_{k|k-1}^T$$  \hspace{1cm} (29)

(7) Calculate the sigma points $X_{i,k|k-1}$

$$X_{i,k|k-1} = S_{k|k-1} \eta_i + \hat{x}_{k|k-1}, i = 1, \ldots, 2n$$  \hspace{1cm} (30)

(8) Propagate the sigma points through measurement function

$$Z_{i,k|k-1} = h\left(X_{i,k|k-1}\right)$$  \hspace{1cm} (31)

(9) Predict the $k$th measurement

$$\hat{z}_{k|k-1} = \sum_{i=0}^{2n} w_i^e Z_{i,k|k-1}$$  \hspace{1cm} (32)

(10) Predict the autocorrelation matrix

$$P_{zz,k|k-1} = \sum_{i=0}^{2n} w_i^e \left(Z_{i,k|k-1} - \hat{z}_{k|k-1}\right)\left(Z_{i,k|k-1} - \hat{z}_{k|k-1}\right)^T + R_k$$  \hspace{1cm} (33)

(11) Predict the cross-correlation matrix
\[
P_{z_i,k|k-1} = \sum_{i=0}^{2n} w_i^f \left( X_{i,i,k|k-1} - \hat{x}_{i,k|k-1} \right) \left( Z_{i,i,k|k-1} - \hat{z}_{i,k|k-1} \right)^T
\]  

(34)

12. Predict the Kalman gain

\[
W_k = P_{z_i,k|k-1} P_{z_i,k|k-1}^{-1}
\]  

(35)

13. Calculate the \( k \)th state estimation and the state error covariance matrix estimation

\[
\hat{x}_{i,k|k} = \hat{x}_{i,k|k-1} + W_k \left( z_k - \hat{z}_{i,k|k-1} \right)
\]  

(36)

\[
P_{i,k|k} = P_{i,k|k-1} - W_k P_{z_i,k|k-1} W_k^T
\]  

(37)

3. If \( k < L \), then set \( k = k + 1 \), go to step 2. If not, end the process. (\( L \) is the expected length of the state estimation)

APPLICATIONS OF UKF IN THE UTC SYSTEM

In Sect. 2 we have deduced the state-space equation of a kind of typical GNSS/INS integrated navigation filter in UTC system. It seems that a good result will be produced as long as the UKF method is applied. However, there are still some problems to be solved owing to the inherent characteristics of the UKF algorithm. The uncertainty in the model and noise can also leads to the decrease of the estimation accuracy, or even worse, the divergence of the final estimation. In Sect. 3 we suppose the covariance matrix of the noise is known in the system model. However, the covariance matrix data is difficult to obtain in most situations. It can be proved that with the aid of the maximum likelihood estimate and the EM algorithm, the robustness and the accuracy of the UKF are improved without clear information of the noise [9]. Based on the theory above, a kind of advanced UKF is proposed in this paper.

The process of the advanced UKF can be summarized as follows:

1. Initialize the values like the step 1 in Sect. 3.2, adding the mean value of noise \( \bar{q}_0 \) and the covariance matrix of noise \( \bar{Q}_0 \).

2. For \( k \geq 1 \) run equation (24) ~ (32) and replace the \( \bar{Q}_{k-1} \) INS (28) as \( \bar{Q}_{k-1} \).

3. Let \( \Delta z_k = z_k - h(\hat{x}_{i,k|k-1}) \) as the residual sequence. If \( \Delta z_k^T \Delta z_k > \text{Str} \left[ E \left( \Delta z_k \Delta z_k^T \right) \right] \), where \( S \geq 1 \) is given, then update the \( P_{i,k|k-1} \) according to

\[
P_{i,k|k-1} = \lambda_k \sum_{i=0}^{2n} w_i^f \left( X_{i,i,k|k-1} - \hat{x}_{i,k|k-1} \right) \left( X_{i,i,k|k-1} - \hat{x}_{i,k|k-1} \right)^T + \bar{Q}_{k-1}
\]  

(38)

\( \lambda_k \) is decided by
\[
\lambda_k = \begin{cases} 
\lambda_0 & \lambda_0 \geq 1 \\
1 & \lambda_0 < 1 
\end{cases}
\]  
(39)

\[
\lambda_0 = \frac{\text{tr} \left( C_k - R_k \right)^T}{\text{tr} \left( \sum_{i=0}^{2n} W_i \left( Z_i, k_{k-1} \right) - \hat{z}_i, k_{k-1} \right) \left( Z_i, k_{k-1} \right) - \hat{z}_i, k_{k-1} \right)^T} 
\]  
(40)

\[
C_k = \begin{cases} 
\Delta z_k \Delta z_k^T & k = 1 \\
\frac{\rho C_k + \Delta z_k \Delta z_k^T}{1 + \rho} & k > 1
\end{cases}
\]  
(41)

The \( \rho = 0.95 \) in this paper. After the update, do the (3) again until \( \Delta z_k^T \Delta z_k \leq \text{Str} \left[ E \left( \Delta z_k \Delta z_k^T \right) \right] \).

(4) Calculating \( \hat{x}_{k|k} \) and \( \hat{P}_{k|k} \) through (33) ~ (37)

(5) Update \( \hat{q}_k \) and \( \hat{Q}_k \) by

\[
\hat{q}_k = (1 - d_{k-1}) \hat{q}_{k-1} + d_{k-1} \left( \hat{x}_{k|k} - f(\hat{x}_{k-1|k-1}) \right) 
\]  
(42)

\[
\hat{Q}_k = (1 - d_{k-1}) \hat{Q}_{k-1} + d_{k-1} \left( W_k w_k w_k^T W_k + P_{k|k-1} - f \cdot P_{k-1|k-1} \cdot f^T \right) 
\]  
(43)

Where \( d_{k-1} = \frac{1 - b}{1 - b_k} \), \( b \) is a constant with \( b \in (0.95, 0.99) \).

RESULTS AND CONCLUSION

Simulations are made in this paper to verify the availability of the advanced UKF in the UTC system. Suppose the state-space equation of the nonlinear system is

\[
\begin{align*}
\dot{x}_k &= 0.5 x_{k-1} + \frac{25 x_{k-1}}{1 + \chi_{k-1}^2} + 8 \cos \left( 1.2 (k - 1) \right) + w_{k-1} \\
\chi_k &= \frac{x_k^2}{2} + v_k
\end{align*}
\]  
(44)

Comparing the estimation state-error of EKF, UKF and AUKF method, we obtain Fig. 2 and Fig. 3. We can explicitly see that the UKF performs better than the EKF and traditional UKF. Fig 3 and Fig .4 show that when the covariance matrix of noise
changes, the AUKF can make more precise estimation than the traditional UKF due to its stronger adaptive capability.

Figure 2. The state estimation of different methods.

Figure 3. The state-error of different methods.

Figure 4. The state-error, $Q_k = \text{diag}\{0.4^2, 0.4^2\}$. 
Figure 5. The state-error, $Q_k = \text{diag}\{ 0.8^2, 0.8^2 \}$.

We can concluded that the UTC navigation system is highly nonlinear, therefore, a nonlinear navigation filter algorithm must be proposed to deal with the problem. The UKF method performs well in the highly nonlinear system when the model are not provided clearly. Furthermore, the AUKF method have stronger capability in adapting the unknown system noise, which can depress the divergence of the system and improve the accuracy of the estimation.

REFERENCES